

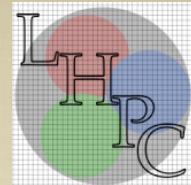
# Nucleon Structure and Lattice QCD

J.W. Negele

PHENIX Spin Fest 2008

BNL

August 8, 2008



# Collaborators

MIT

B. Bistrovic  
J. Bratt  
D. Dolgov  
O. Jahn  
M. F. Lin  
H. Meyer  
A. Pochinsky  
D. Sigaev  
S. Syritsyn

JLab

R. Edwards  
H-W Lin  
D. Richards

William & Mary, JLab

K. Orginos  
Andre Walker-Loud

New Mexico State

M. Engelhardt

Yale

G. Fleming

T. U. Munchen

Ph. Haegler  
B. Musch

Nat. Taiwan U.

W. Schroers  
DESY Zeuthen  
D. Renner

U Cyprus  
C. Alexandrou  
G. Koutsou  
Ph. Leontiou

Athens

A. Tsapalis

ETH, CERN  
Ph. de Forcrand

Julich

Th. Lippert

Wuppertal  
K. Schilling

# Outline

---

- Introduction
  - QCD
  - Lattice Field Theory
  - Computers for lattice QCD
  - Lattice highlights
- Understanding hadron structure
  - Deep inelastic scattering
  - Lattice calculation of nucleon matrix elements
  - Quark distributions
  - Form factors and generalized form factors
  - Transverse structure
  - Origin of nucleon spin
  - Baryon shapes
- Insight into how QCD works
- Summary and future challenges

# QCD

---

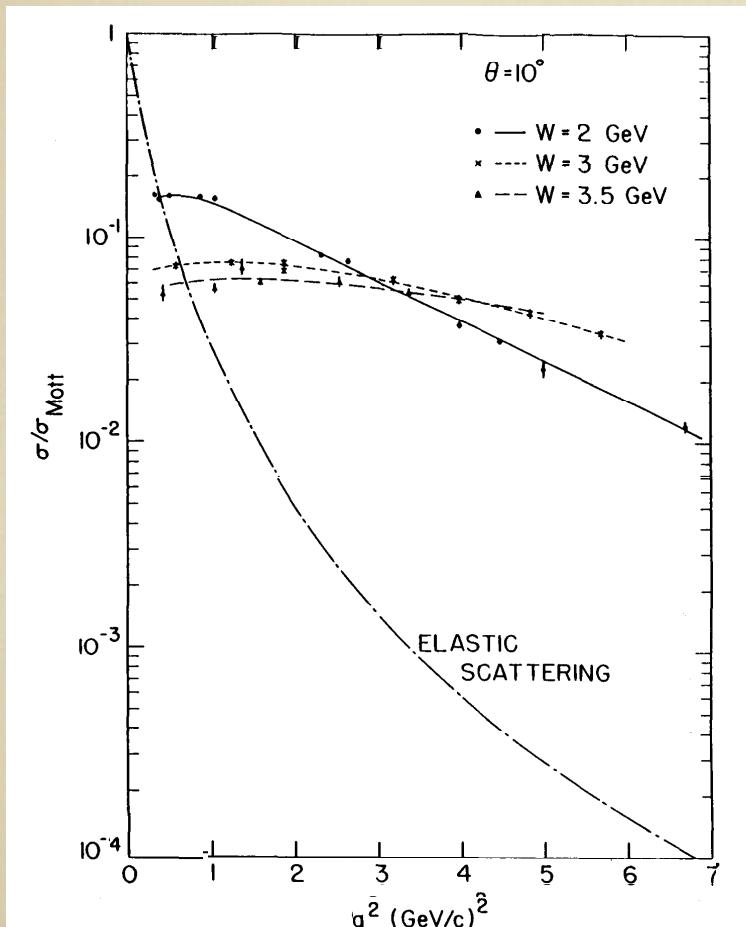
# Introduction - Fundamental Question

---

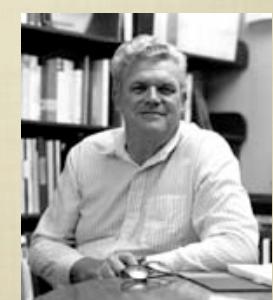
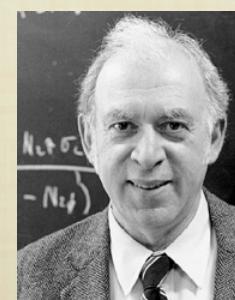
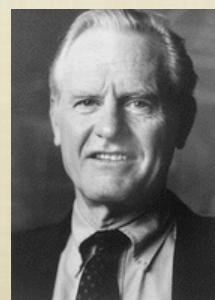
- Hadrons - protons, neutrons and other strongly interacting particles - make up most of the mass of the visible universe
  
- How do we understand the properties and interactions of these basic building blocks of matter from first principles?

# Discovery of Quarks at SLAC

Deep Inelastic electron scattering



1990 Nobel Prize



# DESY

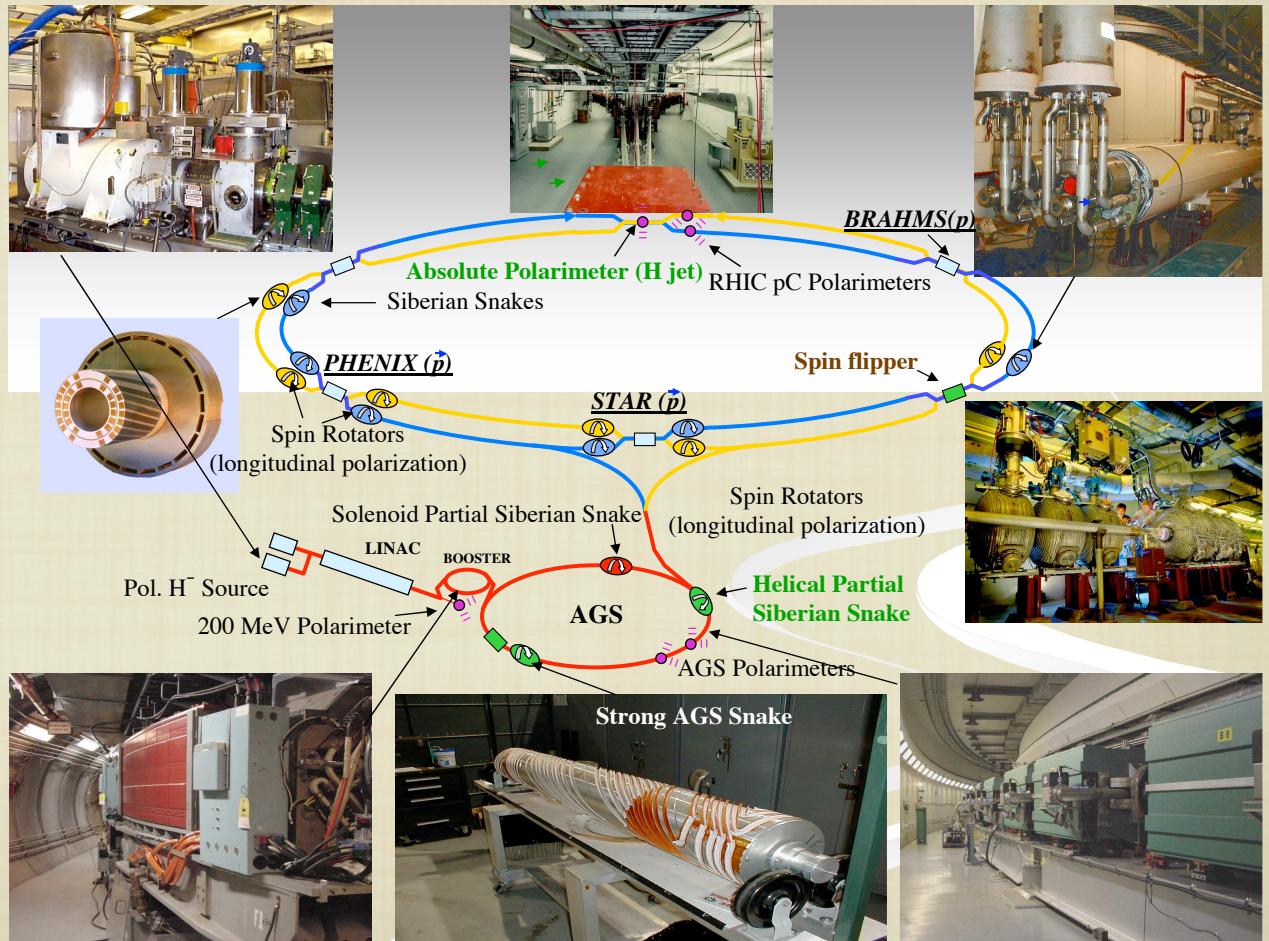
HERA 27.5 GeV electrons on 920 GeV protons

HERMES 27.5 GeV electrons on gas target - Polarization



# RHIC - Spin

250 + 250 GeV polarized protons



# Jefferson Lab Electron Accelerator

Scatter 6 GeV electrons from nucleons  
12 GeV upgrade planned



# Fundamental Question

---

- How do hadrons arise from QCD?
- Lagrangian constrained by Lorentz invariance, gauge invariance and renormalizability:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}^2$$

$$D_\mu = \partial_\mu - igA_\mu \quad F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$$

- Deceptively simple Lagrangian produces amazingly rich and complex structure of strongly interacting matter in our universe

# QCD and Asymptotic Freedom



**David J. Gross**  
Kavli Institute for  
Theoretical  
Physics  
University of  
California, Santa  
Barbara, USA

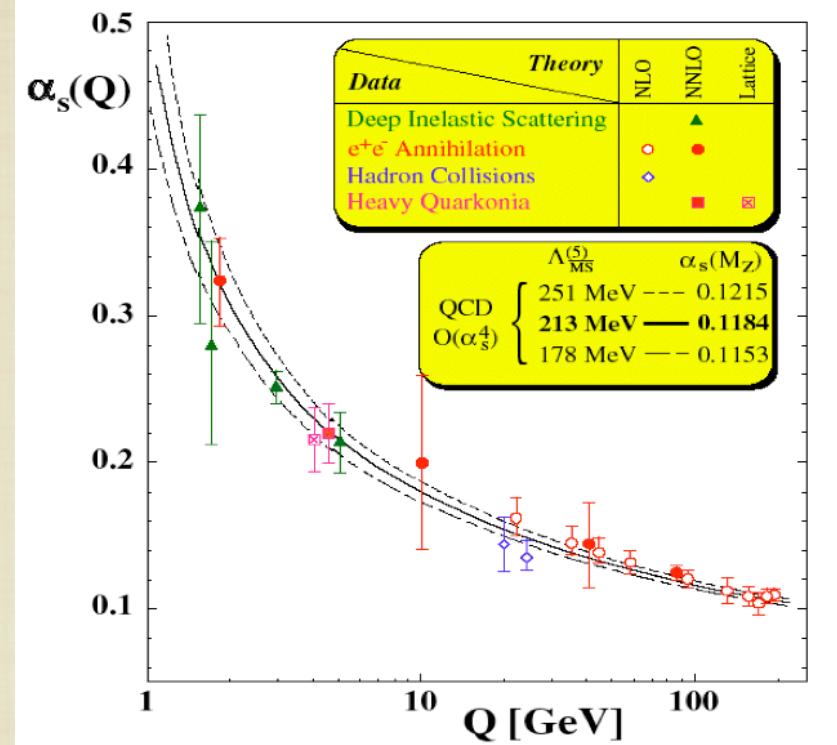


**H. David  
Politzer**  
California  
Institute of  
Technology  
(Caltech),  
Pasadena,  
USA



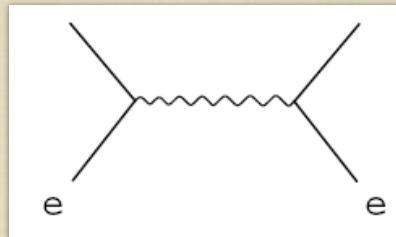
**Frank  
Wilczek**  
Massachusetts  
Institute of  
Technology  
(MIT),  
Cambridge,  
USA

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2004 "for the discovery of asymptotic freedom in the theory of the strong interaction" jointly to David J. Gross, H. David Politzer and Frank Wilczek

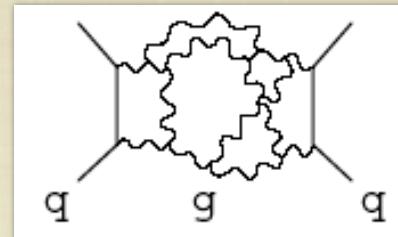


# Nonperturbative QCD

QED



QCD



- Fundamental differences relative to QED
  - Self-interacting: highly nonlinear
  - Interaction increases at large distance: Confinement
  - Interaction decreases at small distance: Asymptotic Freedom
  - Strong coupling:  $\alpha_s \gg \alpha_{em}$
  - Topological excitations
- Solution of nonperturbative QCD
  - Present analytical techniques inadequate
  - Numerical evaluation of path integral on space-time lattice

# Profound differences between hadrons and other many-body systems

---

- Atoms, molecules, nuclei,...
  - Constituents can be removed
  - Exchanged boson generating interaction may be subsumed into static potential
    - photons → Coulomb potential
    - Mesons → N-N potential
  - Most of mass from fermion constituents
- Nucleons
  - Quarks are confined
  - Gluons are essential degrees of freedom
    - Carry half of momentum
    - Nonperturbative topological excitations
  - Most of mass generated by interactions

# Goals

---

- Quantitative calculation of hadron observables from first principles
  - Agreement with experiment
  - Credibility for predictions and guiding experiment
- Insight into how QCD works
  - Mechanisms
    - Origin of nucleon spin and mass
    - Paths that dominate action - instantons
    - Variational wave functions
    - Diquark correlations
  - Dependence on parameters
    - $N_c$ ,  $N_f$ , gauge group,  $m_q$

# How to solve QCD

---

- Analytic methods
  - Perturbation theory
  - Chiral Perturbation theory / Effective field theory
  - String theory techniques to solve somewhat similar theories
- Nonperturbative regime
  - Numerical solution of path integral on space-time lattice

# Lattice Field Theory

---

# Basic Ideas in Lattice QCD

---

Evolution in Euclidean time

$$|\psi\rangle \equiv \sum_n e^{-\beta E_n} C_n |\psi_n\rangle \rightarrow C_0 e^{-\beta E_0} |\psi_0\rangle$$

Lattice Regularization

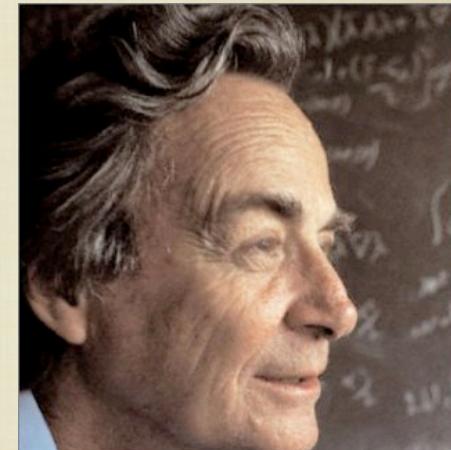
$$\phi(x) \rightarrow \phi(x_n), \quad x_n = n\mathbf{a}$$

Path Integral

$$e^{-\beta \hat{H}} \rightarrow \int D[x(\tau)] e^{-\int_0^\beta d\tau S[x(\tau)]}$$

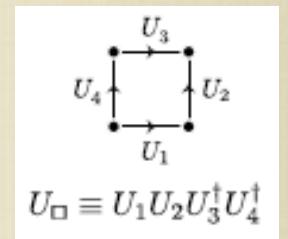
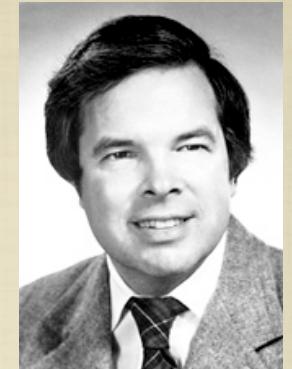
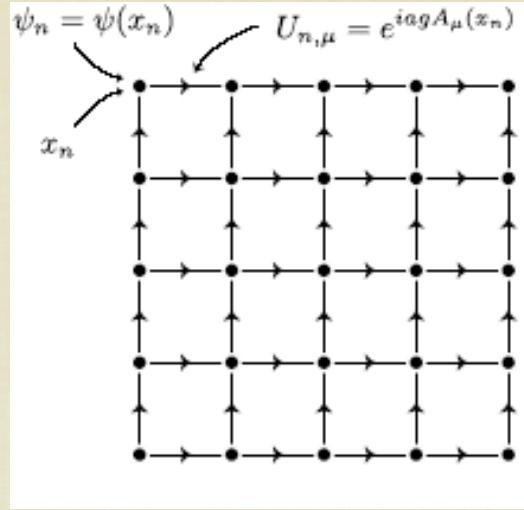
Stochastic Solution

$$\int dx f(x) P(x) = \frac{1}{N} \sum_{x_i \in P} f(x_i) + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$



# Lattice QCD

Wilson put QCD on discrete space-time lattice



$$S(U) = \sum_{\square} \frac{2N}{g^2} (1 - N^{-1} \text{ReTr} U_{\square}) \rightarrow \frac{1}{4} F_{\mu\nu}^2$$

$$\begin{aligned} \bar{\psi} M(U) \psi &= \sum_n [\bar{\psi}_n \psi_n + \kappa (\bar{\psi}_n (1 - \gamma_\mu) U_{n,\mu} \psi_{n+\mu} + \bar{\psi}_{n+\mu} (1 + \gamma_\mu) U_{n,\mu}^\dagger \psi_n)] \\ &\rightarrow \bar{\psi} (\not{\partial} + m + ig \not{A}) \psi \end{aligned}$$

# Lattice QCD

---

$$\langle T e^{-\beta H} \psi \psi \psi \cdots \bar{\psi} \bar{\psi} \bar{\psi} \rangle$$

$$= \frac{1}{Z} \int \mathcal{D}[\psi] \mathcal{D}[\bar{\psi}] \mathcal{D}[A] e^{- \int d^4x [\bar{\psi}(\partial + m + igA)\psi + \frac{1}{4}F_{\mu\nu}^2]} \psi \psi \psi \cdots \bar{\psi} \bar{\psi} \bar{\psi}$$

$$\rightarrow \prod_n \frac{1}{Z} \int d\psi_n d\bar{\psi}_n dU_n e^{-\sum_n [\bar{\psi} M(U) \psi + S(U)]} \psi \psi \psi \cdots \bar{\psi} \bar{\psi} \bar{\psi}$$

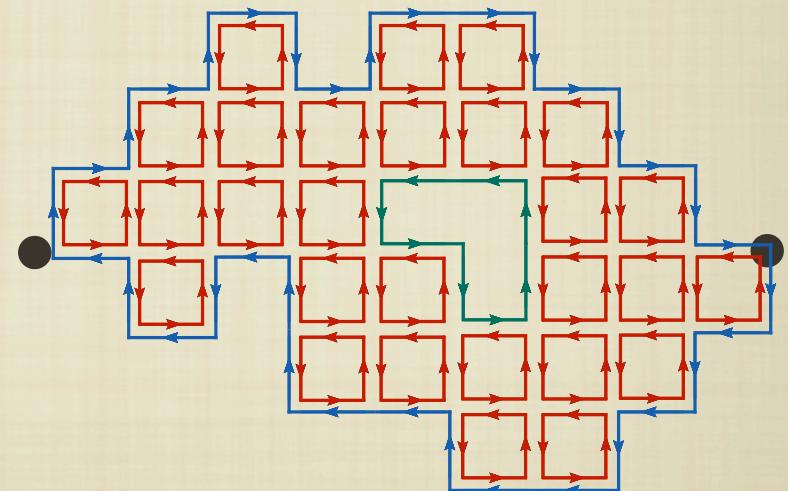
$$= \prod_n \int dU_n \underbrace{\frac{1}{Z} \det M(U) e^{-S(U)}}_{\text{Sample with M.C.}} \sum M^{-1}(U) M^{-1}(U) \cdots M^{-1}(U)$$

$$\rightarrow \frac{1}{N} \sum_{U_i \in \frac{\det M(U)}{Z}}_{i=1}^N M^{-1}(U_i) M^{-1}(U_i) M^{-1}(U_i)$$

# Lattice QCD - summing over paths

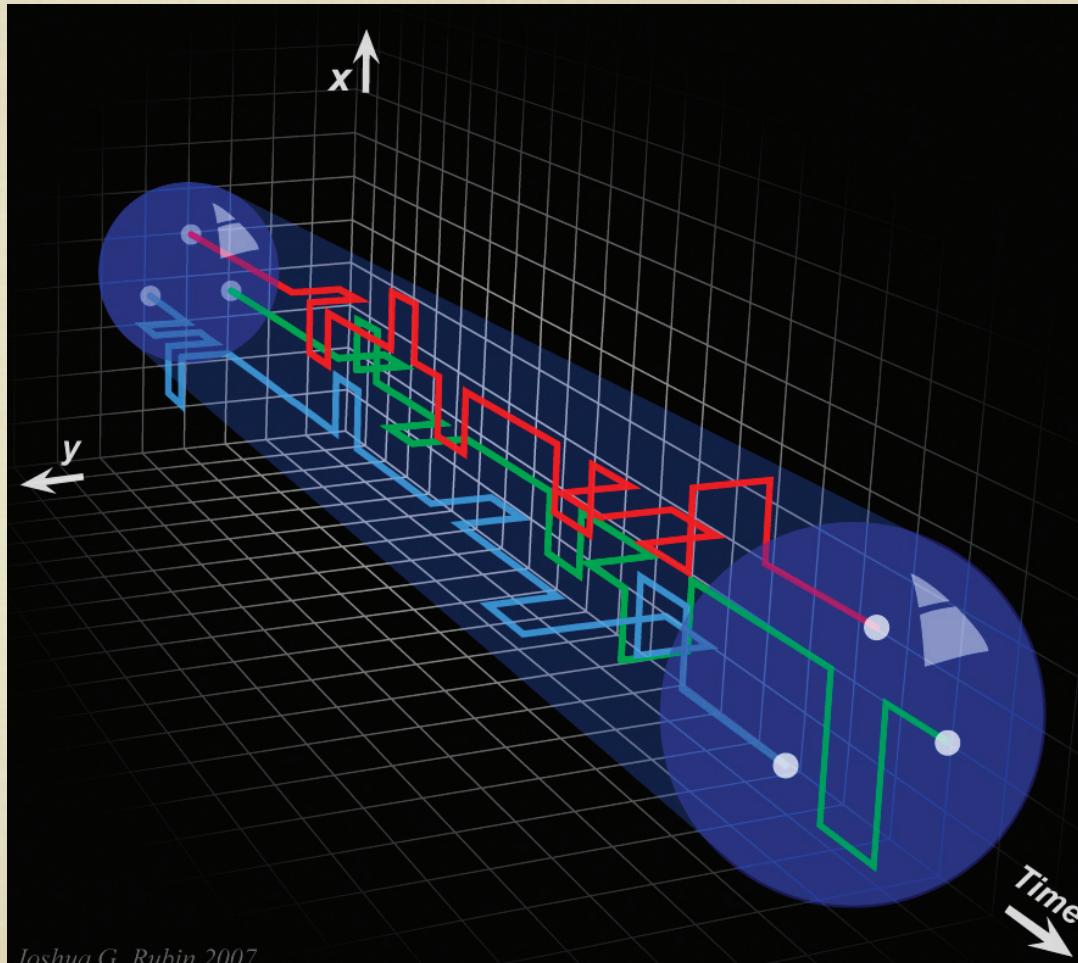
$$\langle T e^{-\beta H} \psi \psi \psi \cdots \bar{\psi} \bar{\psi} \bar{\psi} \rangle = \prod_n \int dU_n \frac{1}{Z} \det M(U) e^{-S(U)} \sum M^{-1}(U) M^{-1}(U) \cdots M^{-1}(U)$$

- $M^{-1} = (I + \kappa U)^{-1}$  connects  $\Psi$ 's with line of  $U$ 's  
Sum over valence quark paths
- $\det M$  generates closed loops of  $U$ 's  
Sum over sea quark excitations
- $S(U)$  tiles with plaquettes  
→ Sum over all gluons
- $32^3 \times 64$  lattices →  $10^8$  gluon variables



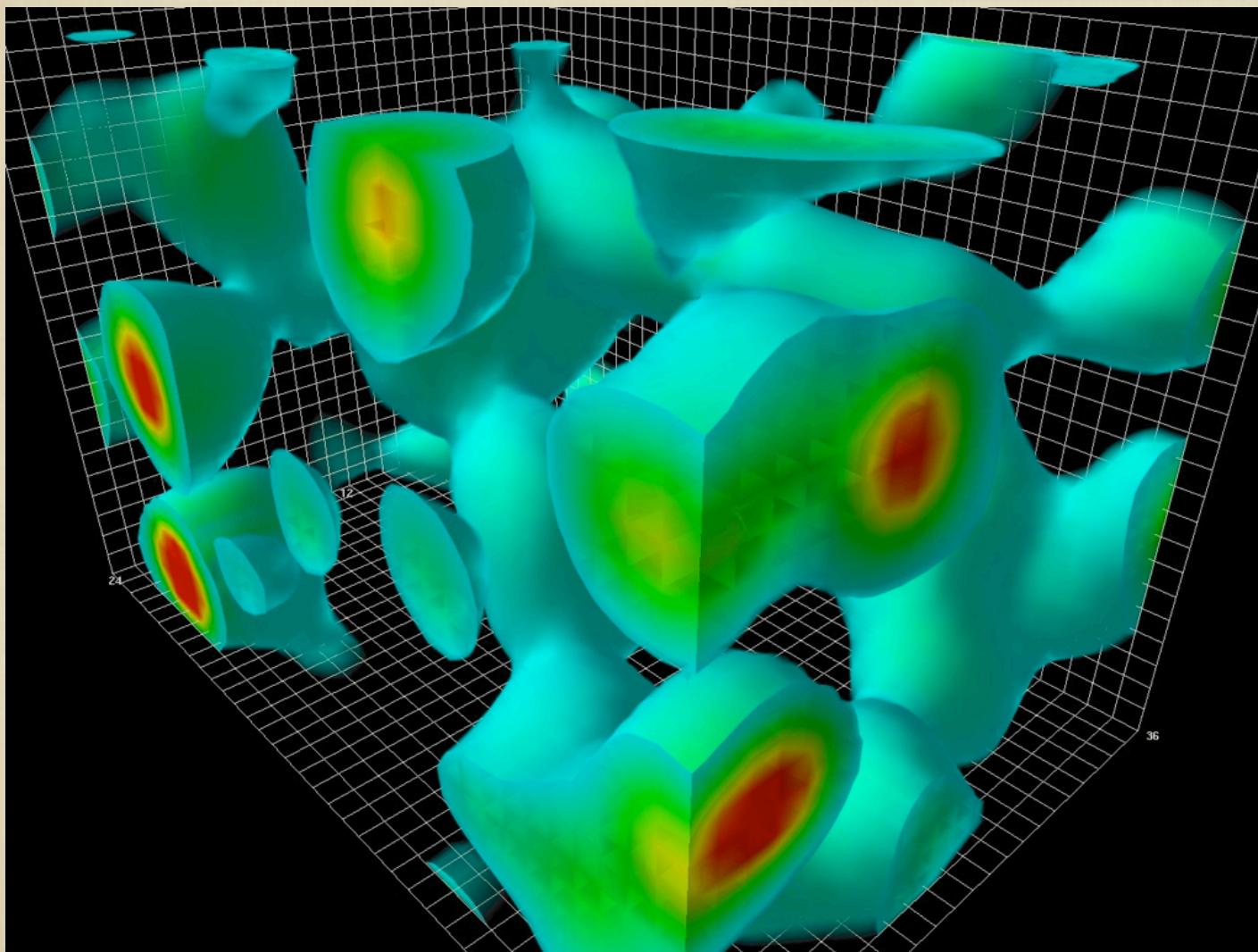
# Quantum Hopscotch

A “typical” time-history for a proton on a lattice



*Joshua G. Rubin 2007*

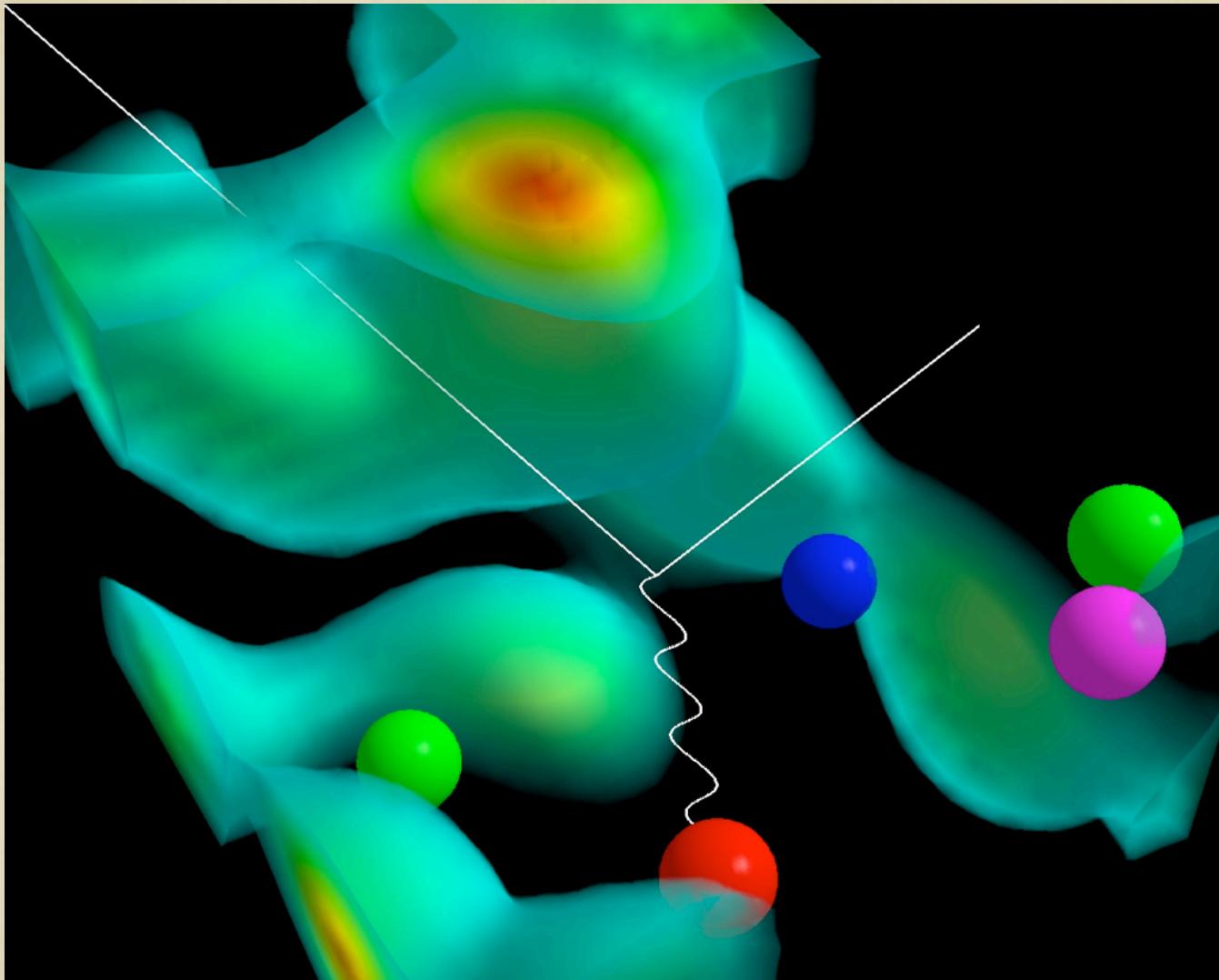
# Gluon fields calculated on the lattice



Visualization by Derek Leinweber

Spin Fest 8-8-08 J.W. Negele

# Quarks interacting with gluons



Visualization by Derek Leinweber

Spin Fest 8-8-08 J.W. Negele

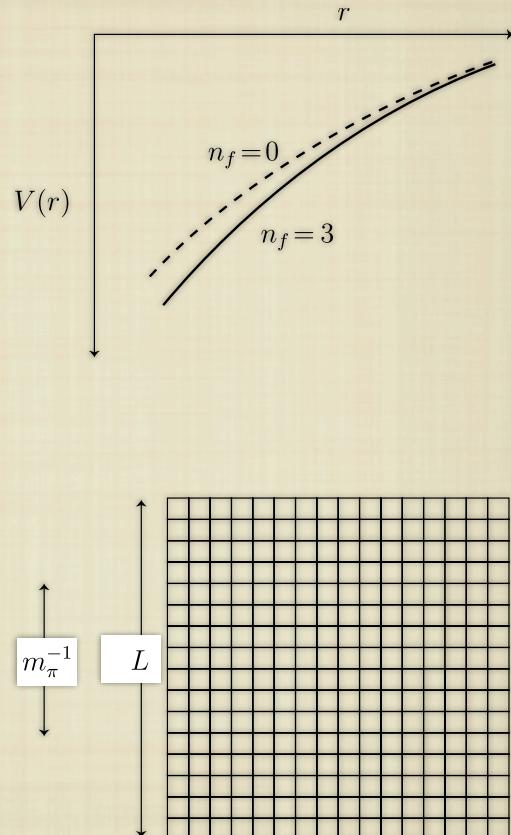
# Computational Issues

- Fermion determinant - Full QCD
- Small lattice spacing
- Small quark mass
- Large lattice volume

$$\frac{1}{m_\pi} \leq \frac{L}{4}$$

L(fm)	$m_\pi$ (Mev)
1.6	500
4.0	200
5.7	140

- Cost  $\sim (m_\pi)^{-7} - (m_\pi)^{-9}$



# Computers for Lattice QCD

---

# Computers for Lattice **QCD**

---

- Multiply 2 16-digit numbers:

$$4.403150586063203 \times 10^{13}$$

$$\times 1.507017275390065 \times 10^{-7}$$

$$= 6.6356239993411342333 \times 10^6$$

- 1 FLoating point OPeration per Second (1 Flops) requires 1 savant

# Computers for Lattice QCD

---

- Megaflops:  $10^6$  Flops  
City with 1 million savants multiplying once a second
- Teraflops:  $10^{12}$  Flops  
1 million cities of 1 million savants
  - US: 10 sustained Teraflops devoted to Lattice QCD
  - World: 50
- Petaflops:  $10^{15}$  Flops
  - LANL - cell based Roadrunner - June, 2008
  - ANL Blue Gene, ORNL Cray 2009
  - NSF Blue Waters at UIUC 2010
- Exaflops:  $10^{18}$  Flops  
1 million planets with 1 million cities of 1 million savants
  - Planning stage

# From QCDOC to Blue Gene

---

- Poster example of societal impact
- Blue Gene/L
  - Al Gara recognized commercial potential, recruited Dong Chen, James Sexton, Pavlos Vranas
  - QCDOC concept, added:  
tree network  
2 proc/node, 2 fpu/proc.



# Lattice QCD Highlights

---

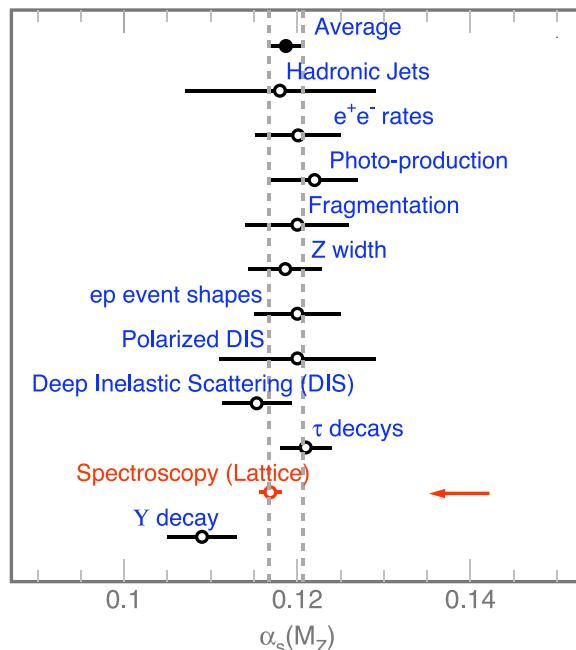
# Heavy Quark Systems

---

The six quarks of the Standard Model:

Flavor	Mass (GeV/c <sup>2</sup> )	Electric Charge (e)
u up	0.004	+2/3
d down	0.008	-1/3
c charm	1.5	+2/3
s strange	0.15	-1/3
t top	176	+2/3
b bottom	4.7	-1/3

# Precision agreement in heavy quark systems



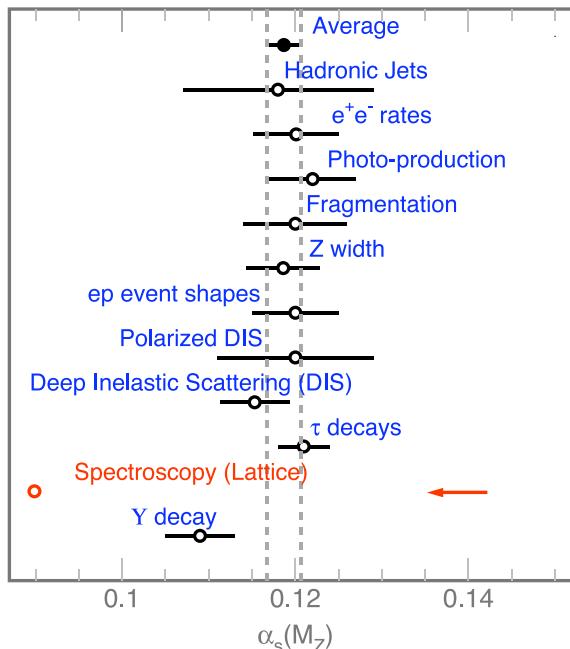
Mason et al, hep-lat/0503005v1 (2005); Particle Data Group (2004 )



$\alpha_s(M_Z)$  from Particle Data Group

# Precision agreement in heavy quark systems

And without light-quark vacuum polarization:

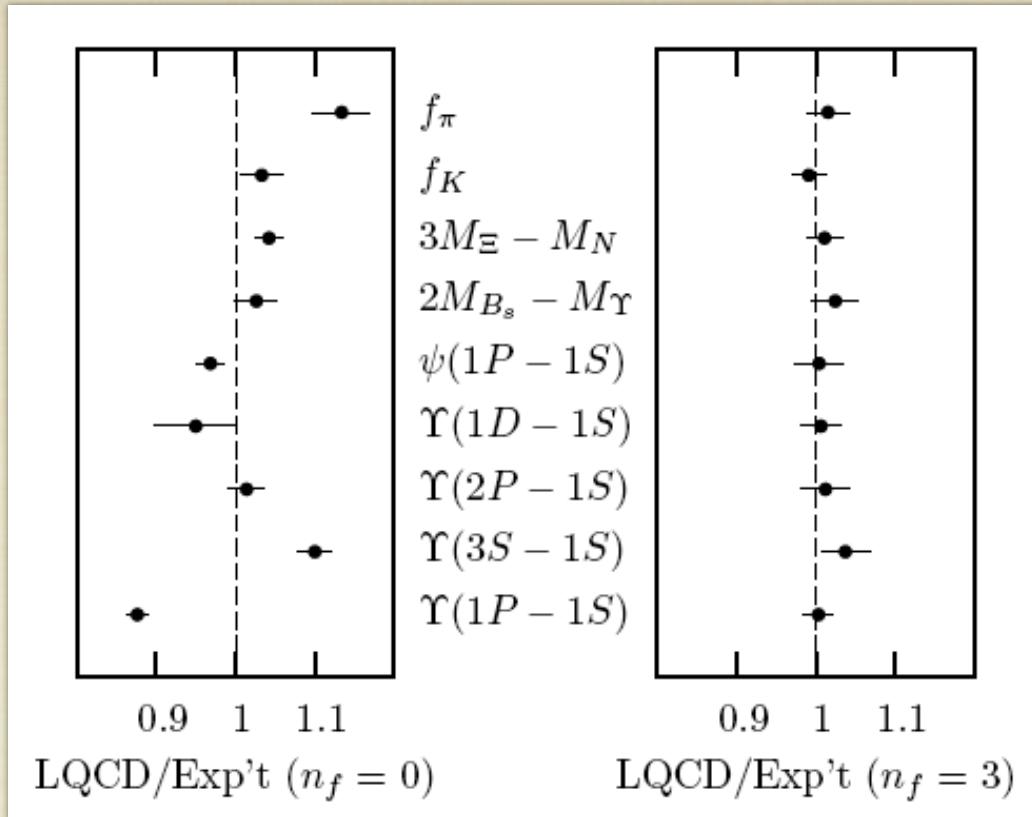


Mason et al, hep-lat/0503005v1 (2005); Particle Data Group (2004)



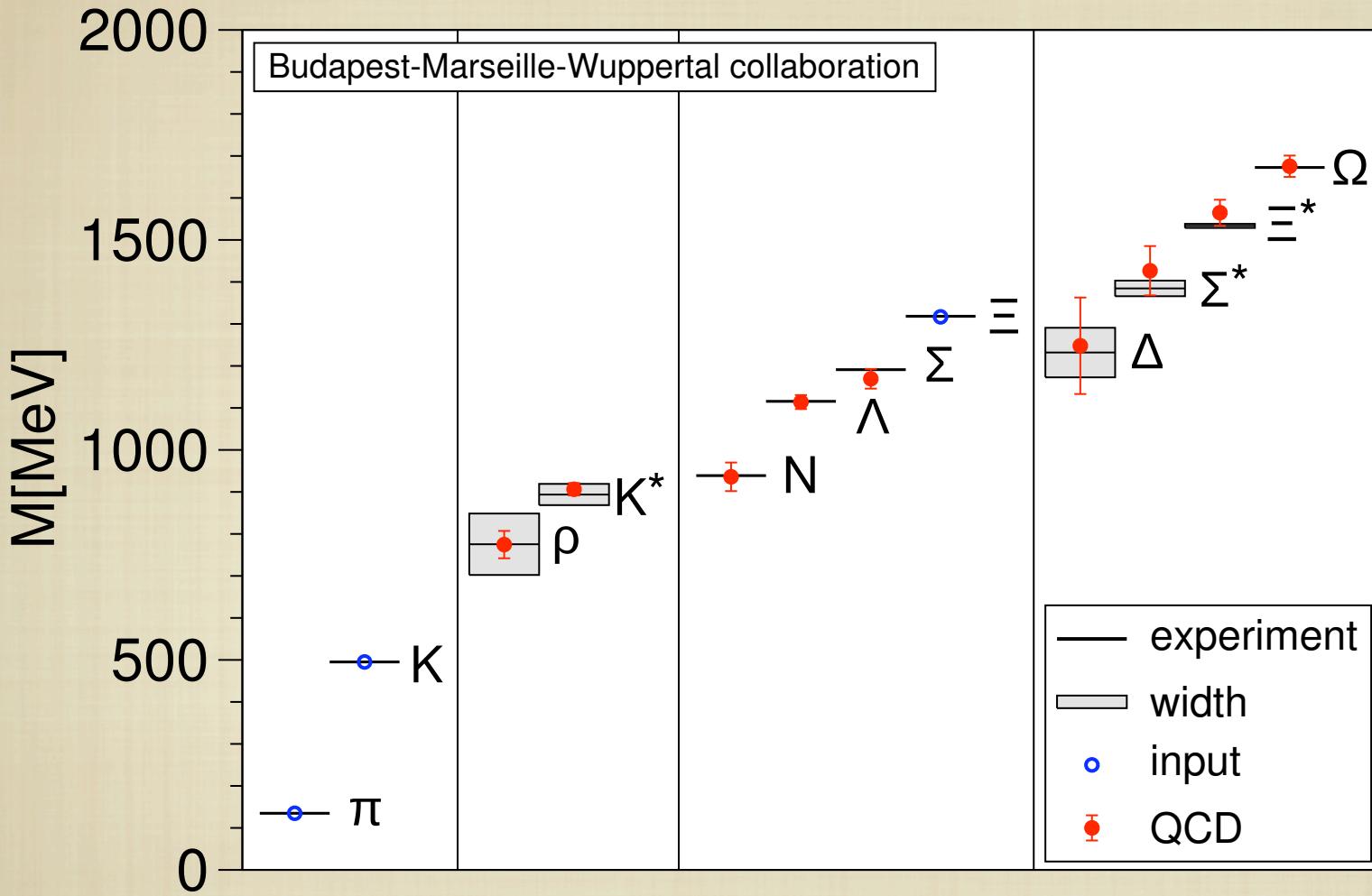
## $\alpha_s(M_Z)$ from Particle Data Group

# Precision agreement in heavy quark systems



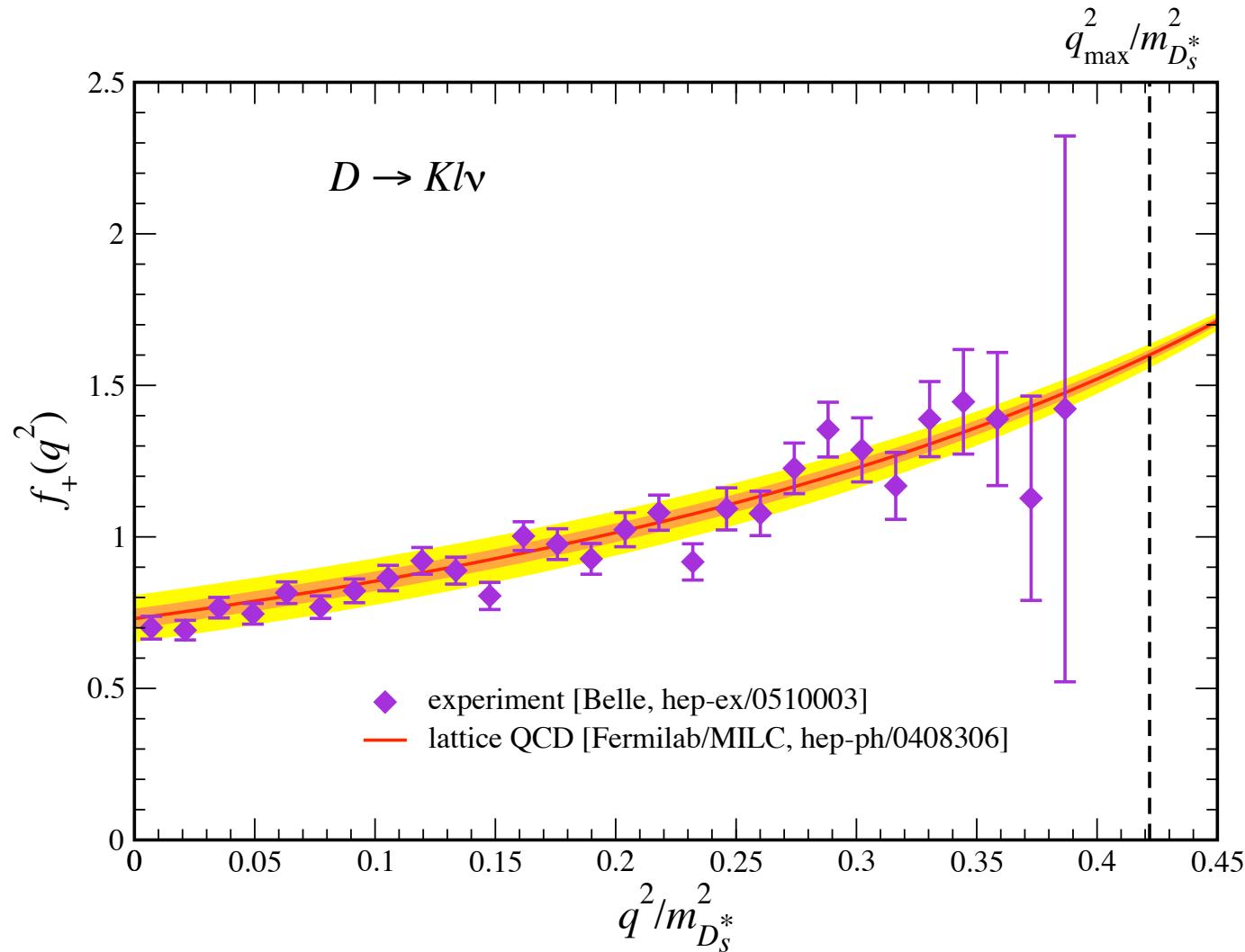
- “Gold Plated Observables” (Davies et. al. hep-lat/0304004)
  - Staggered quarks
  - Asqtad improved action
  - $a = 0.13, 0.09 \text{ fm}$

# Physics - Masses of Hadrons



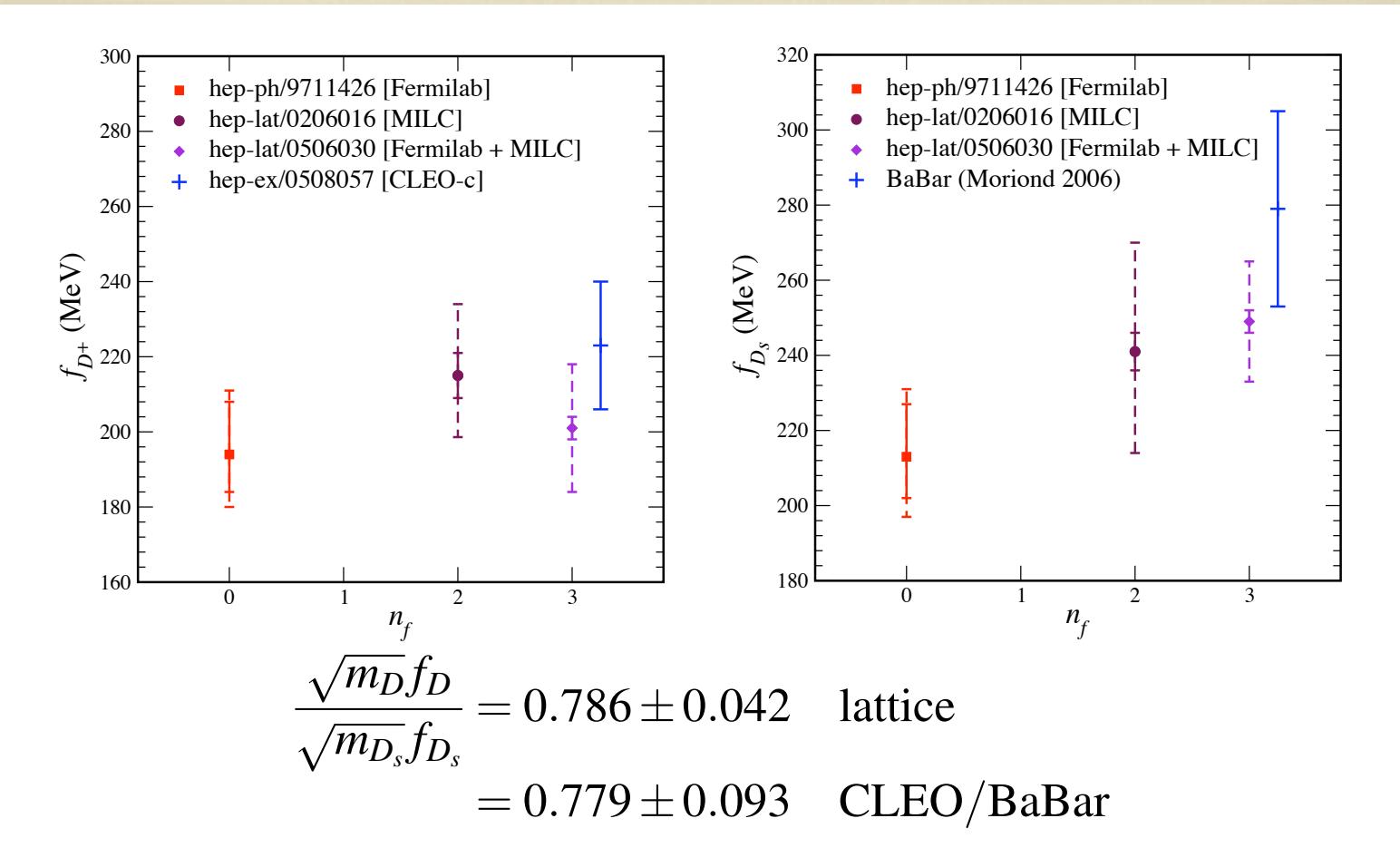
Lattice 2008, BMW collaboration

# Lattice QCD Predictions



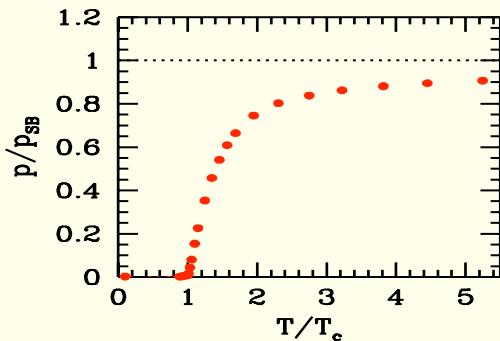
# Lattice QCD Predictions

## D meson decay constants



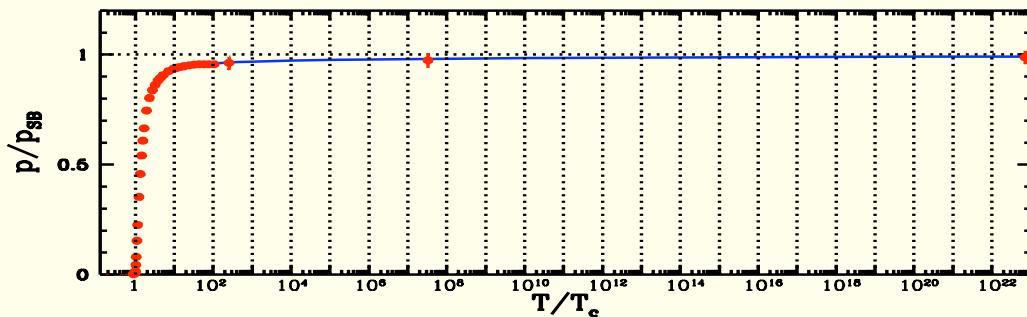
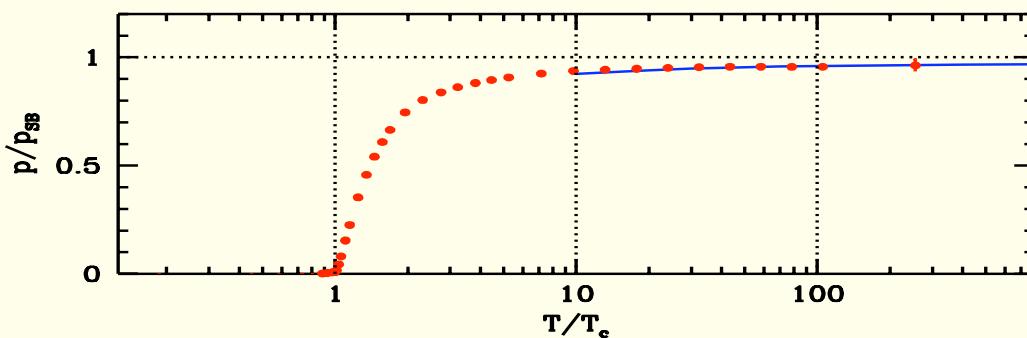
## Mass of $B_c$ meson

# Equation of State of Hadronic Matter

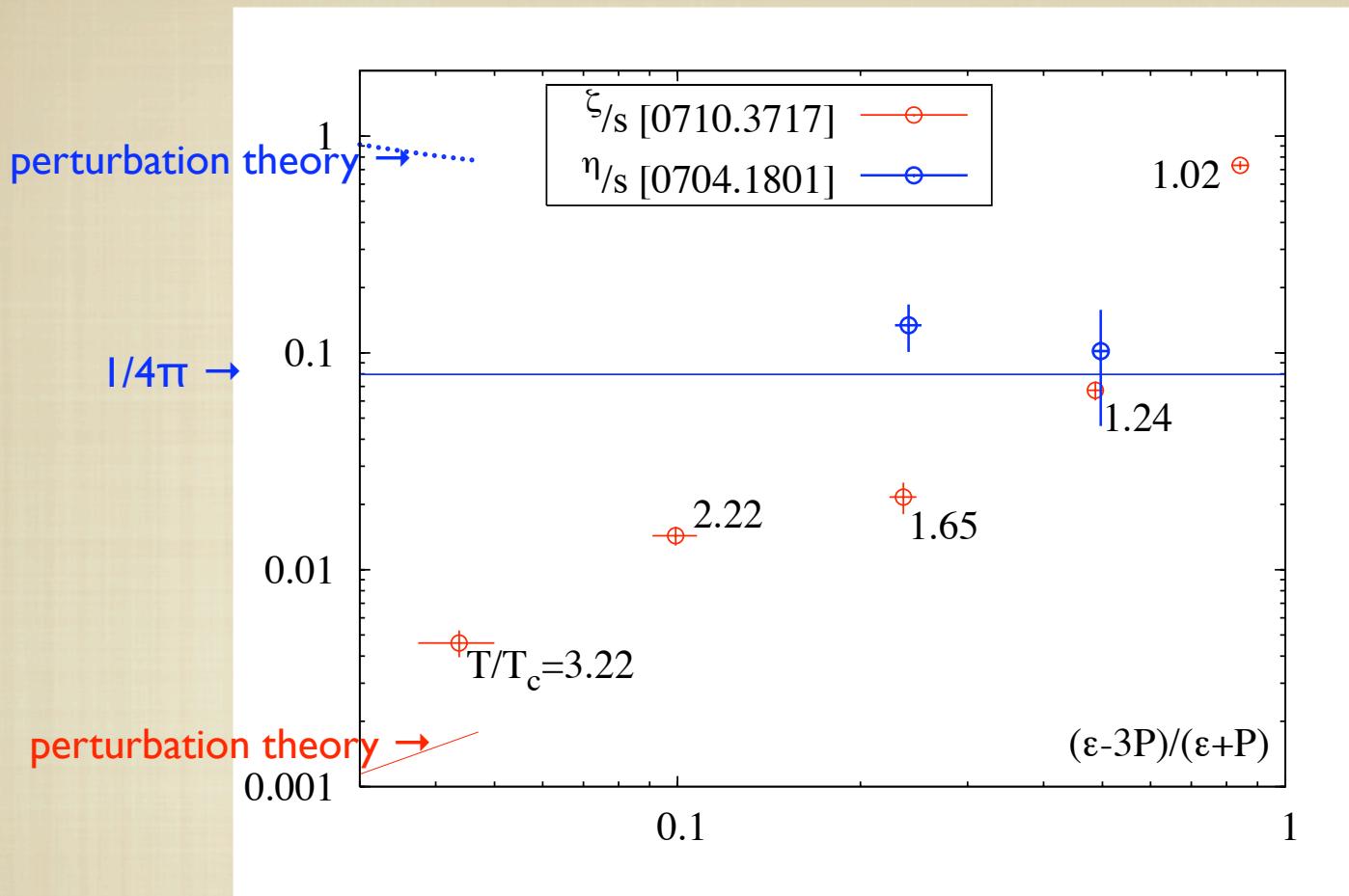


Agreement of lattice QCD  
and perturbative QCD

Z. Fodor, Lattice 2007



# Shear ( $\eta$ ) and bulk ( $\zeta$ ) viscosity in SU(3) lattice QCD



Shear ( $\eta$ ) and bulk ( $\zeta$ ) viscosity in the deconfined gluon plasma  
compared with the  $1/4\pi$  obtained in strongly coupled Super Yang Mills

H, Meyer hep-lat/0704.1801, hep-lat/0710.3717

# High Energy Scattering and Nucleon Structure

---

# Hadron structure revealed by high energy scattering

---

- High energy scattering measures correlation functions along light cone
  - Asymptotic freedom: reaction theory perturbative
  - Unambiguous measurement of operators in light cone frame
  - Must think about physics on light cone
- Parton distribution  $q(x)$  gives longitudinal momentum distribution of light-cone wave function
- Generalized parton distribution  $q(x, r_\perp)$  gives transverse spatial structure of light-cone wave function

# Parton and generalized parton distributions

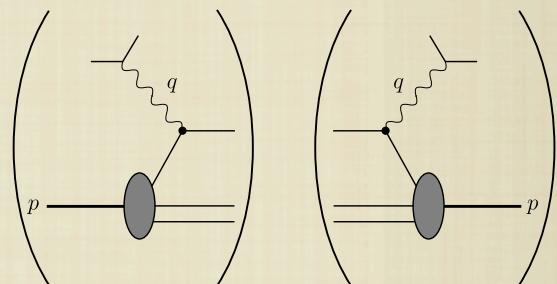
High energy scattering: light-cone correlation function    ( $\lambda = p^+ x^-$ )

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) \not{p} \mathcal{P} e^{-ig \int_{-\lambda/2}^{\lambda/2} d\alpha n \cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

Deep inelastic scattering: diagonal matrix element

$$\langle P | \mathcal{O}(x) | P \rangle = q(x)$$

$$[\not{p} \rightarrow \not{p} \gamma_5 : \Delta q(x)]$$

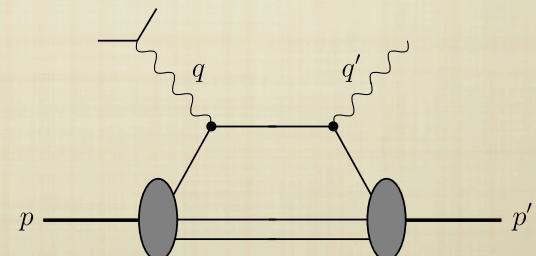


Deeply virtual Compton scattering: off-diagonal matrix element

$$\langle P' | \mathcal{O}(x) | P \rangle = \langle \gamma \rangle H(x, \xi, t) + \frac{i\Delta}{2m} \langle \sigma \rangle E(x, \xi, t)$$

$$\Delta = P' - P, \quad t = \Delta^2, \quad \xi = -n \cdot \Delta / 2$$

$$[\not{p} \rightarrow \not{p} \gamma_5 : \tilde{E}(x, \xi, t), \tilde{H}(x, \xi, t)]$$



# Moments of parton distributions

Expansion of  $\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \not{p} \mathcal{P} e^{-ig \int_{-\lambda/2}^{\lambda/2} d\alpha n \cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$

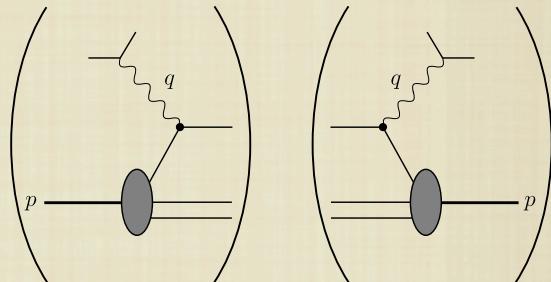
Generates tower of twist-2 operators

$$\mathcal{O}_q^{\{\mu_1 \mu_2 \dots \mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} \psi_q$$

Diagonal matrix element

$$\langle P | \mathcal{O}_q^{\{\mu_1 \mu_2 \dots \mu_n\}} | P \rangle \sim \int dx x^{n-1} q(x)$$

$$[\not{p} \rightarrow \not{p} \gamma_5 : \Delta q(x)]$$



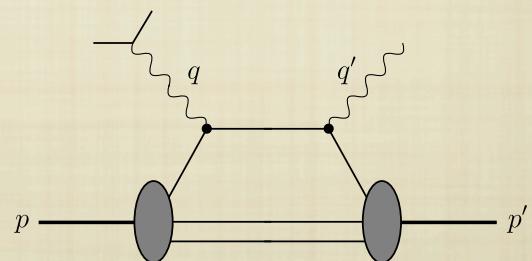
Off-diagonal matrix element

$$\langle P' | \mathcal{O}_q^{\{\mu_1 \mu_2 \dots \mu_n\}} | P \rangle \rightarrow A_{ni}(t), B_{ni}(t), C_{n0}(t)$$

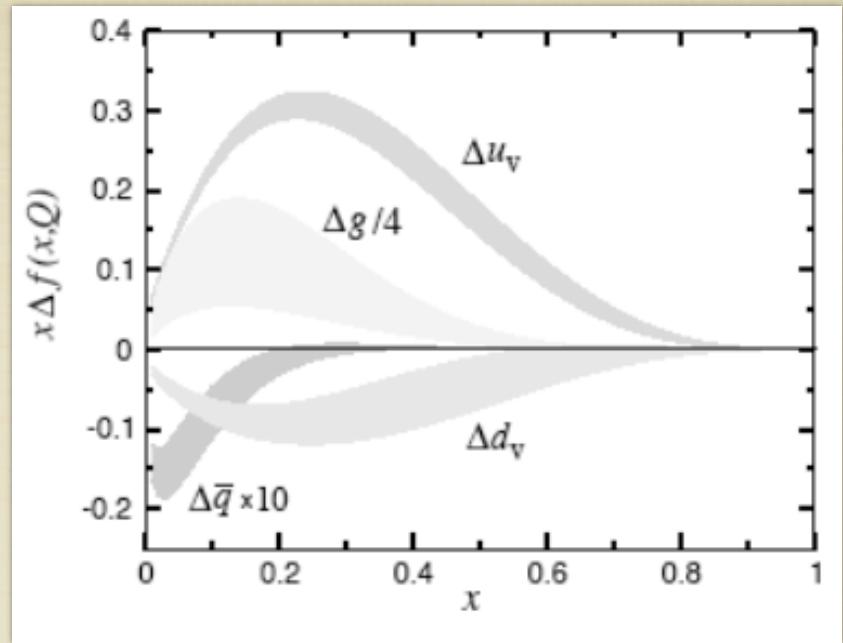
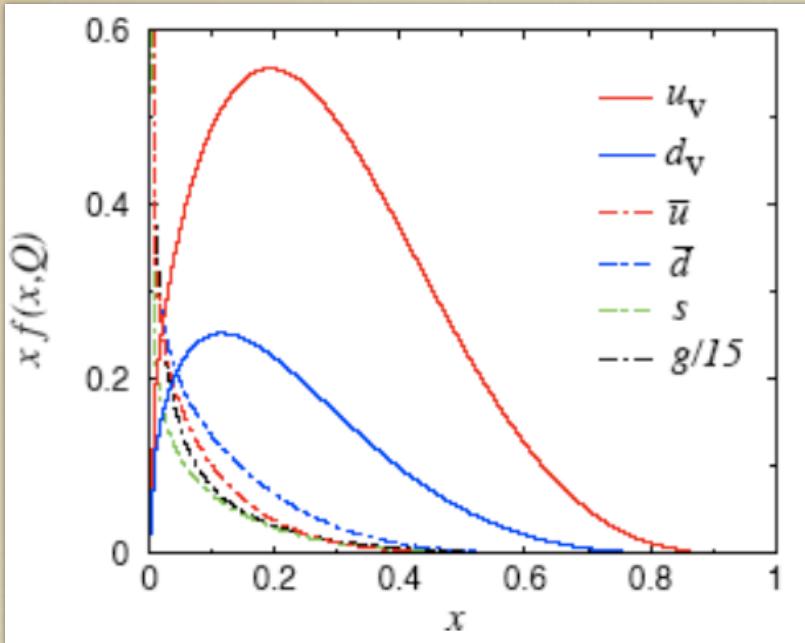
$$\int dx x^{n-1} H(x, \xi, t) \sim \sum \xi^i A_{ni}(t) + \xi^n C_{n0}(t)$$

$$\int dx x^{n-1} E(x, \xi, t) \sim \sum \xi^i B_{ni}(t) - \xi^n C_{n0}(t)$$

$$[\not{p} \rightarrow \not{p} \gamma_5 : \tilde{A}_{ni}(t), \tilde{B}_{ni}(t)]$$



# Moments of parton distributions



$$\begin{aligned}
 \langle p | \bar{\psi} \gamma_\mu D_{\mu_1} \cdots D_{\mu_n} \psi | p \rangle &\rightarrow \langle x^n \rangle_q &= \int_0^1 dx x^n [q(x) + (-1)^{(n+1)} \bar{q}(x)] \\
 \langle p | \bar{\psi} \gamma_5 \gamma_\mu D_{\mu_1} \cdots D_{\mu_n} \psi | p \rangle &\rightarrow \langle x^n \rangle_{\Delta q} &= \int_0^1 dx x^n [\Delta q(x) + (-1)^{(n)} \Delta \bar{q}(x)] \\
 \langle p | \bar{\psi} \gamma_5 \sigma_{\mu\nu} D_{\mu_1} \cdots D_{\mu_n} \psi | p \rangle &\rightarrow \langle x^n \rangle_{\delta q} &= \int_0^1 dx x^n [\delta q(x) + (-1)^{(n+1)} \delta \bar{q}(x)]
 \end{aligned}$$

where  $q = q_\uparrow + q_\downarrow$ ,  $\Delta q = q_\uparrow - q_\downarrow$ ,  $\delta q = q_\top + q_\perp$ ,

# Lattice operators: irreducible representations of hypercubic group with minimal operator mixing and minimal non-zero momentum components

$\langle x \rangle_q^{(a)}$	$6_3^+$	$\bar{\psi} \gamma_{\{1} \overleftrightarrow{D}_4 \} \psi$
$\langle x \rangle_q^{(b)}$	$3_1^+$	$\bar{\psi} \gamma_4 \overleftrightarrow{D}_4 \psi - \frac{1}{3} \sum_{i=1}^3 \bar{\psi} \gamma_i \overleftrightarrow{D}_i \psi$
$\langle x^2 \rangle_q$	$8_1^-$	$\bar{\psi} \gamma_{\{1} \overleftrightarrow{D}_1 \overleftrightarrow{D}_4 \} \psi - \frac{1}{2} \sum_{i=2}^3 \gamma_{\{i} \overleftrightarrow{D}_i \overleftrightarrow{D}_4 \} \psi$
$\langle x^3 \rangle_q$	$2_1^+$	$\bar{\psi} \gamma_{\{1} \overleftrightarrow{D}_1 \overleftrightarrow{D}_4 \overleftrightarrow{D}_4 \} \psi + \bar{\psi} \gamma_{\{2} \overleftrightarrow{D}_2 \overleftrightarrow{D}_3 \overleftrightarrow{D}_3 \} \psi - \{3 \leftrightarrow 4\}$
$\langle 1 \rangle_{\Delta q}$	$4_4^+$	$\bar{\psi} \gamma^5 \gamma_3 \psi$
$\langle x \rangle_{\Delta q}^{(a)}$	$6_3^-$	$\bar{\psi} \gamma^5 \gamma_{\{1} \overleftrightarrow{D}_3 \} \psi$
$\langle x \rangle_{\Delta q}^{(b)}$	$6_3^-$	$\bar{\psi} \gamma^5 \gamma_{\{3} \overleftrightarrow{D}_4 \} \psi$
$\langle x^2 \rangle_{\Delta q}$	$4_2^+$	$\bar{\psi} \gamma^5 \gamma_{\{1} \overleftrightarrow{D}_3 \overleftrightarrow{D}_4 \} \psi$
$\langle 1 \rangle_{\delta q}$	$6_1^+$	$\bar{\psi} \gamma^5 \sigma_{34} \psi$
$\langle x \rangle_{\delta q}$	$8_1^-$	$\bar{\psi} \gamma^5 \sigma_{3\{4} \overleftrightarrow{D}_{1\}} \psi$
$d_1$	$6_1^+$	$\bar{\psi} \gamma^5 \gamma_{[3} \overleftrightarrow{D}_{4]} \psi$
$d_2$	$8_1^-$	$\bar{\psi} \gamma^5 \gamma_{[1} \overleftrightarrow{D}_{\{3} \overleftrightarrow{D}_{4\}} \psi$

# Lattice Calculation of Nucleon M. E.

---

# Domain wall quarks on a staggered sea

---

- $\mathcal{O}(a^2)$  Tadpole improved staggered sea quarks (Asqtad)
  - Economical entre to chiral regime
  - MILC 2+1 flavor lattices with large  $L$ , small  $m_\pi$  publicly available
- Domain wall valence quarks
  - Chiral symmetry to within controlled  $m_{res}$
  - Avoids operator mixing
  - $\mathcal{O}(a^2)$
  - Conserved 5-d axial current facilitates renormalization
- Mixed action ChPT Chen, O'Connell, Walker-Loud, arXiv: 0706.00035
  - One-loop results have continuum chiral behavior with low energy constants containing perturbative  $a$ -dependent corrections

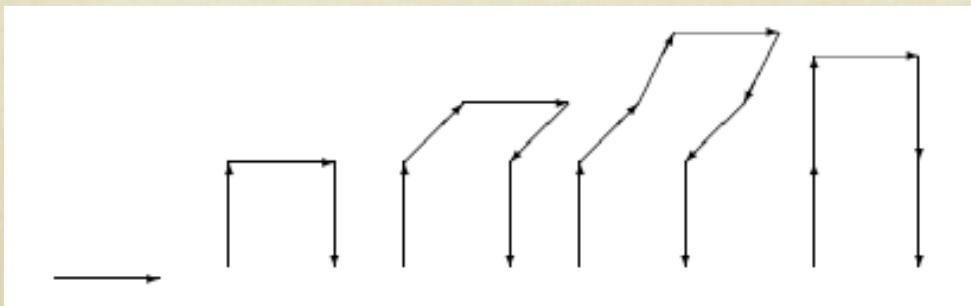
# Numerical calculations

- MILC Asqtad configurations  $N_F = 2+1$ ,  $a = 0.125 \text{ fm}$
- Domain wall valence quarks
  - $L_S = 16$ ,  $M_5 = 1.7$
  - Valence quark mass tuned to Asqtad Goldstone pion mass.
  - Recent improvement: **Factor 8 increase in # measurements**

$m_\pi$	# configs	Vol	L (fm)	# measurements	
758	423	$20^3$	2.5	423	
688	348	$20^3$	2.5	348	
597	561	$20^3$	2.5	561	
495	477	$20^3$	2.5	477	
356	628	$20^3$	2.5	628	<b>5024</b>
353	274	$28^3$	3.5	274	<b>2192</b>
293	464	$20^3$	2.5		<b>3712</b>

# Asqtad Action: $\mathcal{O}(a^2)$ perturbatively improved

- Symansik improved glue
  - $S_g(U) = C_0 W^{1 \times 1} + C_1 W^{1 \times 2} + C_2 W^{\text{cube}}$
- Smeared staggered fermions  $S_f(V, U)$ 
  - Fat links remove taste changing gluons
  - Tadpole improved



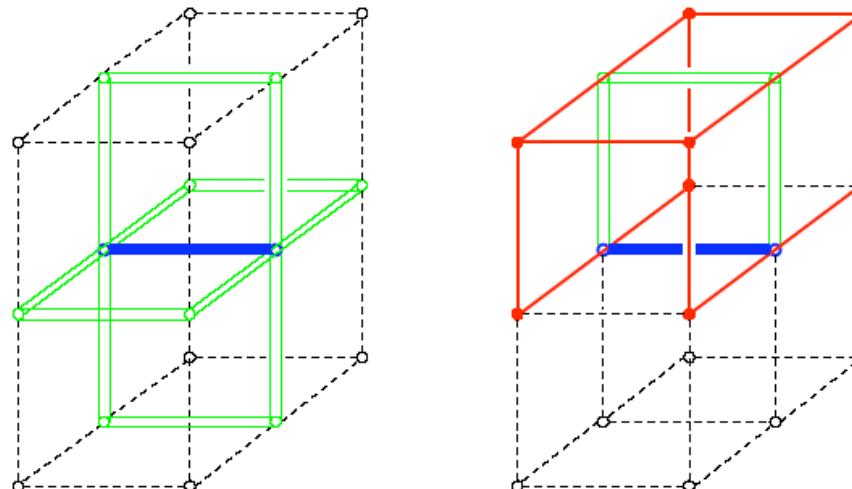
# HYP Smearing

- Three levels of SU(3) projected blocking within hypercube
- Minimize dislocations - important for DW fermions

$$V_{i,\mu} = \text{Proj}_{SU(3)}[(1 - \alpha_1)U_{i,\mu} + \frac{\alpha_1}{6} \sum_{\pm v \neq \mu} \tilde{V}_{i,v;\mu} \tilde{V}_{i+\hat{v},\mu;v} \tilde{V}_{i+\hat{\mu},v;\mu}^\dagger],$$

$$\tilde{V}_{i,\mu;v} = \text{Proj}_{SU(3)}[(1 - \alpha_2)U_{i,\mu} + \frac{\alpha_2}{4} \sum_{\pm \rho \neq v, \mu} \tilde{V}_{i,\rho;v} \tilde{V}_{i+\hat{\rho},\mu;\rho;v} \tilde{V}_{i+\hat{\mu},\rho;v;\mu}^\dagger],$$

$$\tilde{V}_{i,\mu;v\rho} = \text{Proj}_{SU(3)}[(1 - \alpha_3)U_{i,\mu} + \frac{\alpha_3}{2} \sum_{\pm \eta \neq \rho, v, \mu} U_{i,\eta} U_{i+\hat{\eta},\mu} U_{i+\hat{\mu},\eta}^\dagger].$$



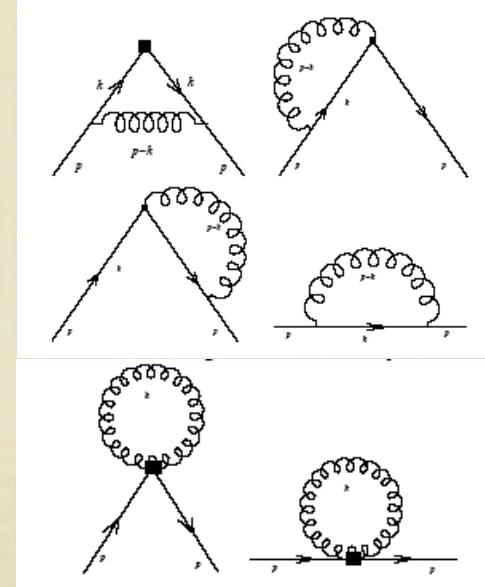
# Perturbative renormalization

$$O_i^{\overline{MS}}(Q^2) = \sum_j \left( \delta_{ij} + \frac{g_0^2}{16\pi^2} \frac{N_c^2 - 1}{2N_c} \left( \gamma_{ij}^{\overline{MS}} \log(Q^2 a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}}) \right) \right) \cdot O_j^{LATT}(a^2)$$

HYP smeared domain wall fermions - B. Bistrovic

$$Z_O = \frac{Z_O^{pert}}{Z_A^{pert}} Z_A^{nonpert} \quad \text{Evolve to } Q^2 = 4 \text{ GeV}^2$$

operator	$H(4)$	HYP
$\bar{q}[\gamma_5]q$	$1_1^\pm$	0.981
$\bar{q}[\gamma_5]\gamma_\mu q$	$4_4^\mp$	0.976
$\bar{q}[\gamma_5]\sigma_{\mu\nu}q$	$6_1^\mp$	0.992
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$6_3^\pm$	0.979
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$3_1^\pm$	0.975
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu}D_{\alpha\}}q$	$8_1^\mp$	0.988
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu}D_{\alpha\}}q$	mixing	$1.88 \times 10^{-3}$
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu}D_{\alpha\}}q$	$4_2^\mp$	0.987
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu}D_{\alpha}D_{\beta\}}q$	$2_1^\pm$	0.993
$\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}}q$	$8_1^\pm$	0.994
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}q$	$6_1^\mp$	0.982
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}D_{\alpha\}}q$	$8_1^\pm$	0.959



# Statistics for hadron structure

---

- Signal to noise degrades as pion mass decreases

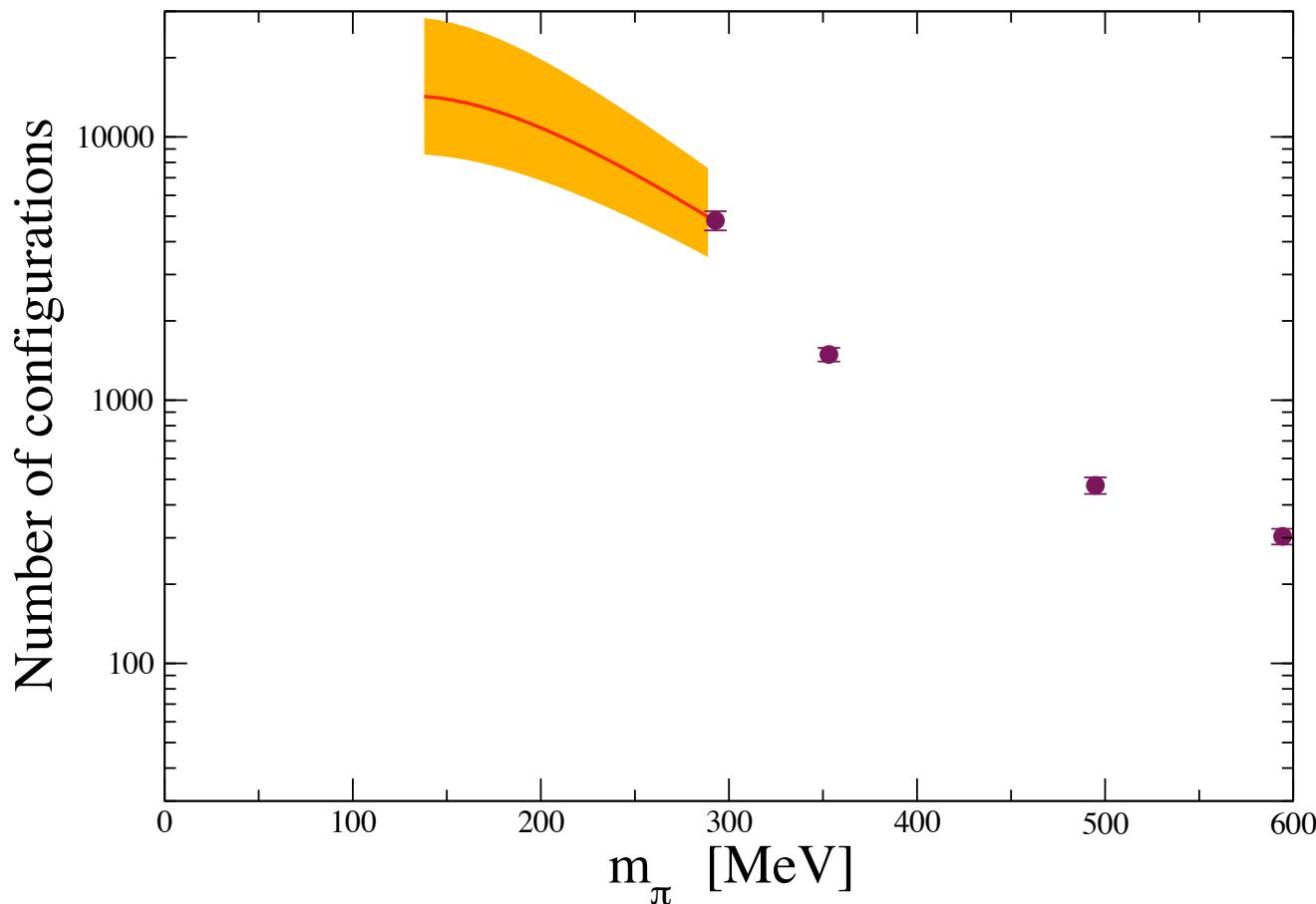
$$\begin{aligned}\frac{\text{Signal}}{\text{Noise}} &= \frac{\langle J(t)J(0) \rangle}{\frac{1}{\sqrt{N}} \sqrt{\langle |J(t)J(0)|^2 \rangle - (\langle J(t)J(0) \rangle)^2}} \\ &\sim \frac{Ae^{-M_N t}}{\frac{1}{\sqrt{N}} \sqrt{Be^{-3m_\pi t} - Ce^{-2M_N t}}} \\ &\sim \sqrt{N} D e^{-(M_N - \frac{3}{2}m_\pi)t}\end{aligned}$$

- Due to different overlap of nucleon and 3 pions also have volume dependence:  $\sqrt{V}$
- Kostas Orginos analyzed signal/noise correlation functions for mixed action data

# Required Measurements

Measurements required for 3% accuracy at T=10

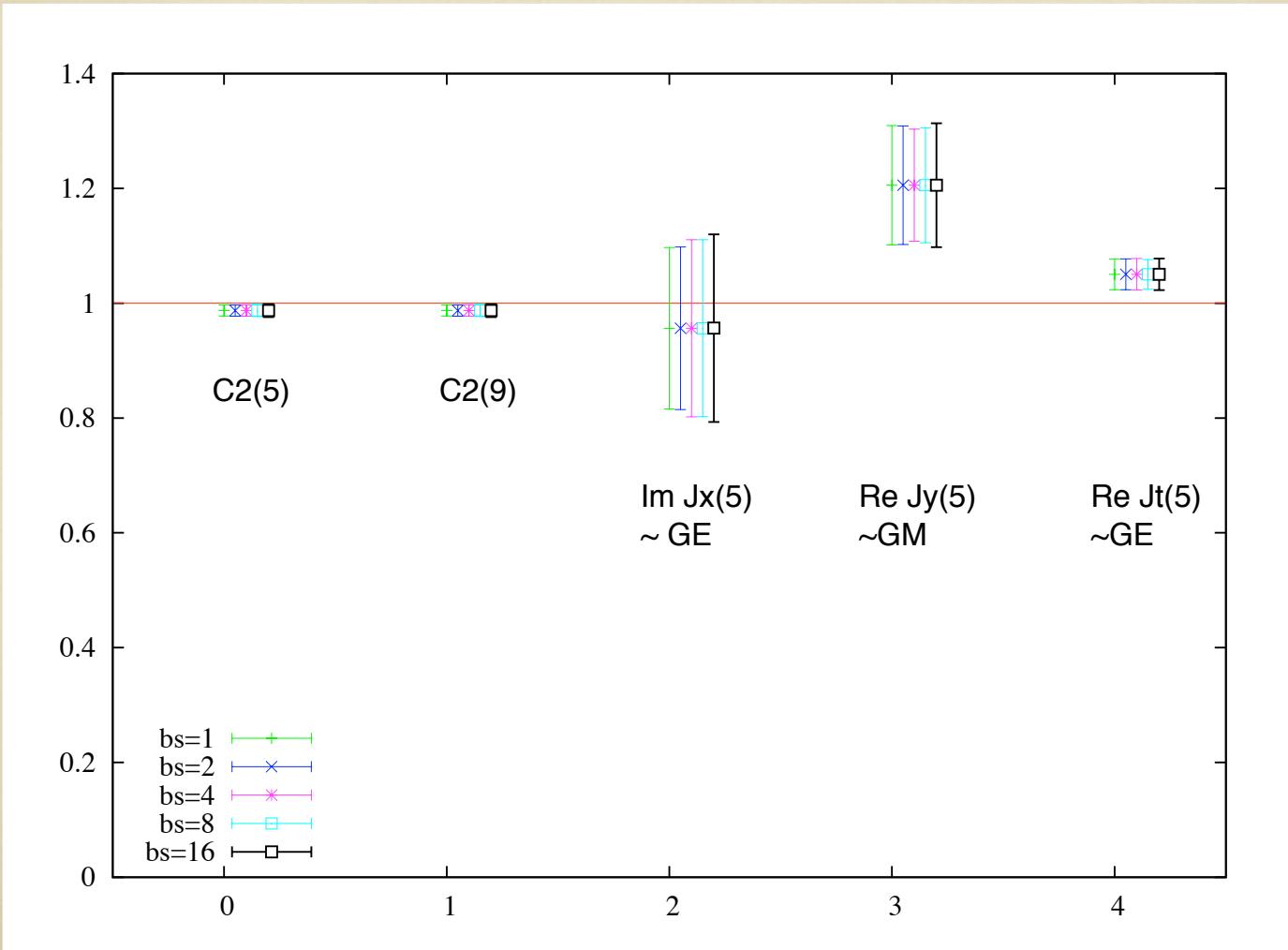
May need significantly more



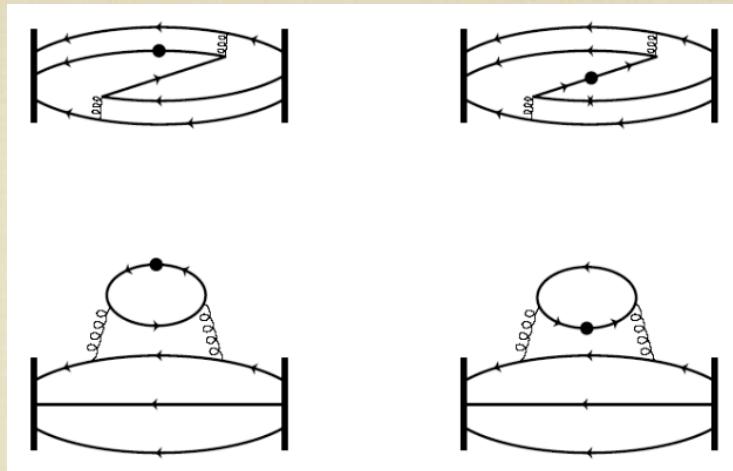
# Statistical independence of measurements

Jackknife binning of correlation functions and matrix elements

Sergey Syritsyn



# Hadron matrix elements on the lattice



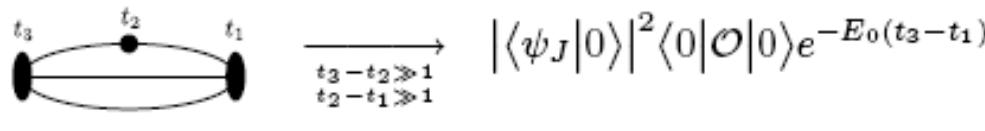
- Measure  $\langle \mathcal{O} \rangle$  for  $m_q, a, L$
- Connected diagrams
- Disconnected diagrams (cancel for  $\langle \mathcal{O} \rangle_u - \langle \mathcal{O} \rangle_d$ )
- Extrapolate  $m_q, a, L$

# Matrix elements on the lattice

$J^\dagger$  : Current with quantum numbers of proton

$|\psi_J\rangle = J^\dagger |\Omega\rangle$  Trial function

$$\langle TJ(t_3) \mathcal{O}(t_2) J^\dagger(t_1) \rangle = \sum_{m,n} \langle \psi_J | n \rangle \langle n | \mathcal{O} | m \rangle \langle m | \psi_J \rangle e^{-E_n(t_3-t_2)-E_m(t_2-t_1)}$$



Normalize:

$$\langle TJ(t_3) J^\dagger(t_1) \rangle = \sum_n |\langle \psi_J | n \rangle|^2 e^{-E_n(t_3-t_1)}$$

$$\xrightarrow[t_3-t_1 \gg 1]{} |\langle \psi_J | 0 \rangle|^2 e^{-E_0(t_3-t_1)}$$

⇒

$$\langle 0 | \mathcal{O} | 0 \rangle = \frac{\langle J \mathcal{O} J^\dagger \rangle}{\langle J J^\dagger \rangle} = \frac{\text{Diagram with one loop}}{\text{Diagram with two loops}}$$

# Overdetermined system for form factors

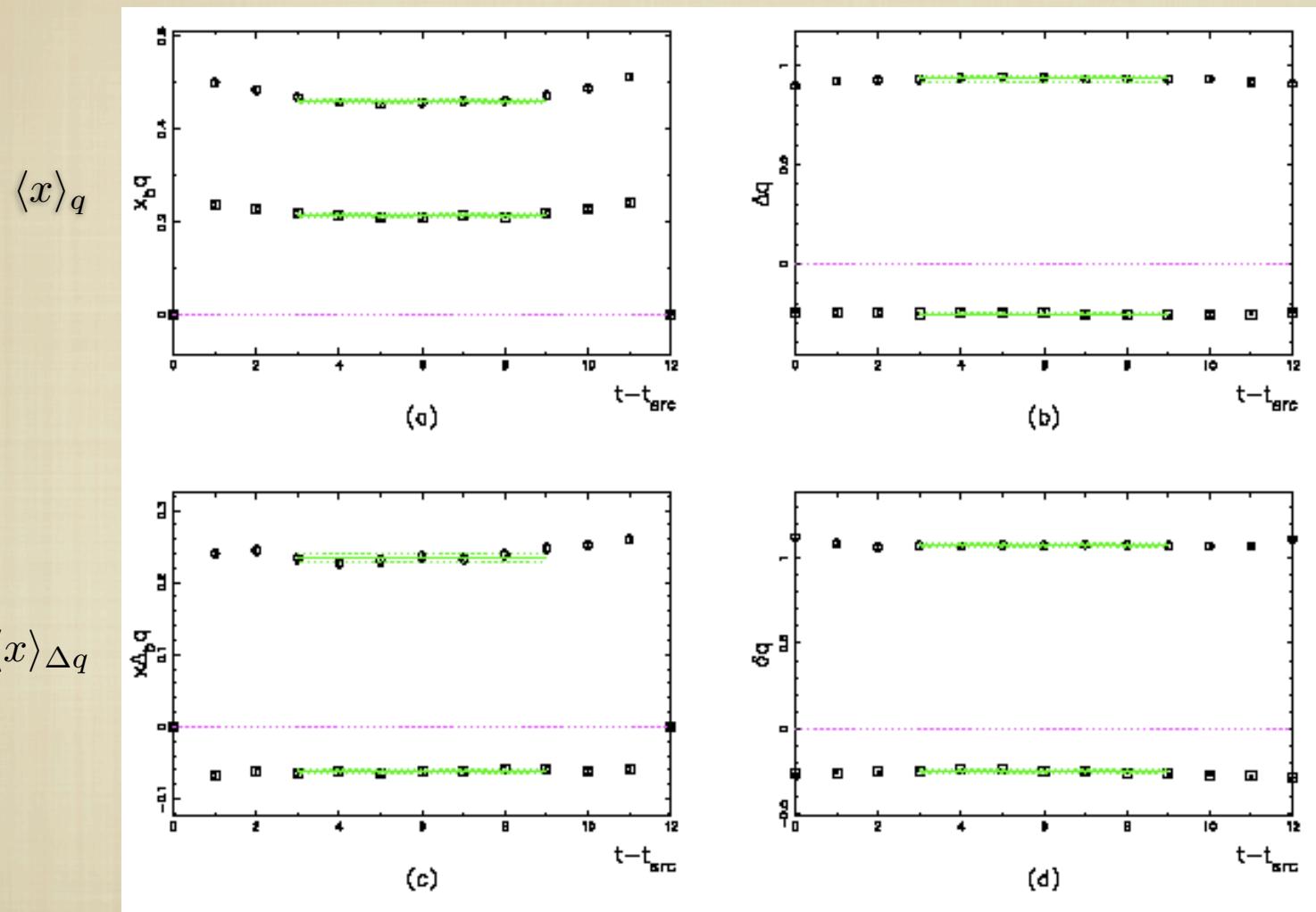
Calculate ratio

$$R_{\mathcal{O}}(\tau, P', P) = \frac{C_{\mathcal{O}}^{\text{3pt}}(\tau, P', P)}{C^{\text{2pt}}(\tau_{\text{snk}}, P')} \left[ \frac{C^{\text{2pt}}(\tau_{\text{snk}} - \tau + \tau_{\text{src}}, P) C^{\text{2pt}}(\tau, P') C^{\text{2pt}}(\tau_{\text{snk}}, P')}{C^{\text{2pt}}(\tau_{\text{snk}} - \tau + \tau_{\text{src}}, P') C^{\text{2pt}}(\tau, P) C^{\text{2pt}}(\tau_{\text{snk}}, P)} \right]^{1/2}$$

Schematic form

$$\begin{aligned}\langle \mathcal{O}_i^{\text{cont}} \rangle &= \sum_j a_{ij} \mathcal{F}_j \\ \langle \mathcal{O}_i^{\text{cont}} \rangle &= \sqrt{E'E} \sum_j Z_{ij} \bar{R}_j \\ \bar{R}_i &= \frac{1}{\sqrt{E'E}} \sum_{jk} Z_{ij}^{-1} a_{jk} \mathcal{F}_k \\ &\equiv \sum_j a'_{ij} \mathcal{F}_j.\end{aligned}$$

# Plateaus for operators



# Axial Charge

---

# Nucleon axial charge in full lattice QCD

---

□ Why  $g_A$ ?

□ Matrix element of axial current       $A_\mu = \bar{q} \gamma_\mu \gamma_5 \frac{\vec{\tau}}{2} q$

$$\langle N(p+q) | A_\mu | N(p) \rangle = \bar{u}(p+q) \frac{\vec{\tau}}{2} [g_A(q^2) \gamma_\mu \gamma_5 + g_P(q^2) q_\mu \gamma_5] u(p)$$

$$g_A(0) = 1.2695 \pm 0.0029$$

□ Adler Weisberger       $g_A^2 - 1 \sim \int (\sigma_{\pi^+ p} - \sigma_{\pi^- p})$

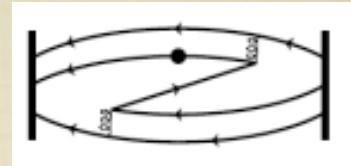
□ Goldberger Treiman       $g_A \rightarrow f_\pi g_{\pi NN}/M_N$

□ Spin content       $\langle 1 \rangle_{\Delta q} = \int_0^1 dx [\Delta q(x) + \Delta \bar{q}(x)]$

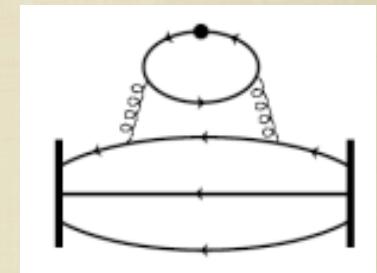
$$g_A = \langle 1 \rangle_{\Delta u} - \langle 1 \rangle_{\Delta d} \quad \Sigma = \langle 1 \rangle_{\Delta u} + \langle 1 \rangle_{\Delta d} + \langle 1 \rangle_{\Delta s}$$

# Nucleon axial charge

- Gold-Plated observable



- Accurately measured



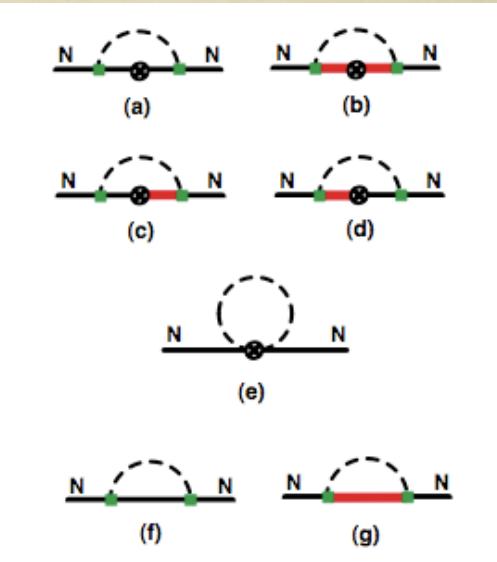
- No disconnected diagrams

- Chiral perturbation theory for  $g_A(m_\pi^2, V)$

- Renormalization - 5-d conserved current

# Nucleon Axial Charge

- Chiral perturbation theory  $g_A(m_\pi^2, V)$ 
  - Beane and Savage hep-ph/0404131
  - Detmold and Lin hep-lat/0501007
- I-loop theory has 6 parameters
  - Fix  $f_\pi, m_\Delta - m_N, g_{\Delta N}$  (0.3% error)
  - Fit  $g_A, g_{\Delta\Delta}, C$
  - Result  $g_A(m_\pi = 140) = 1.212 \pm 0.084$



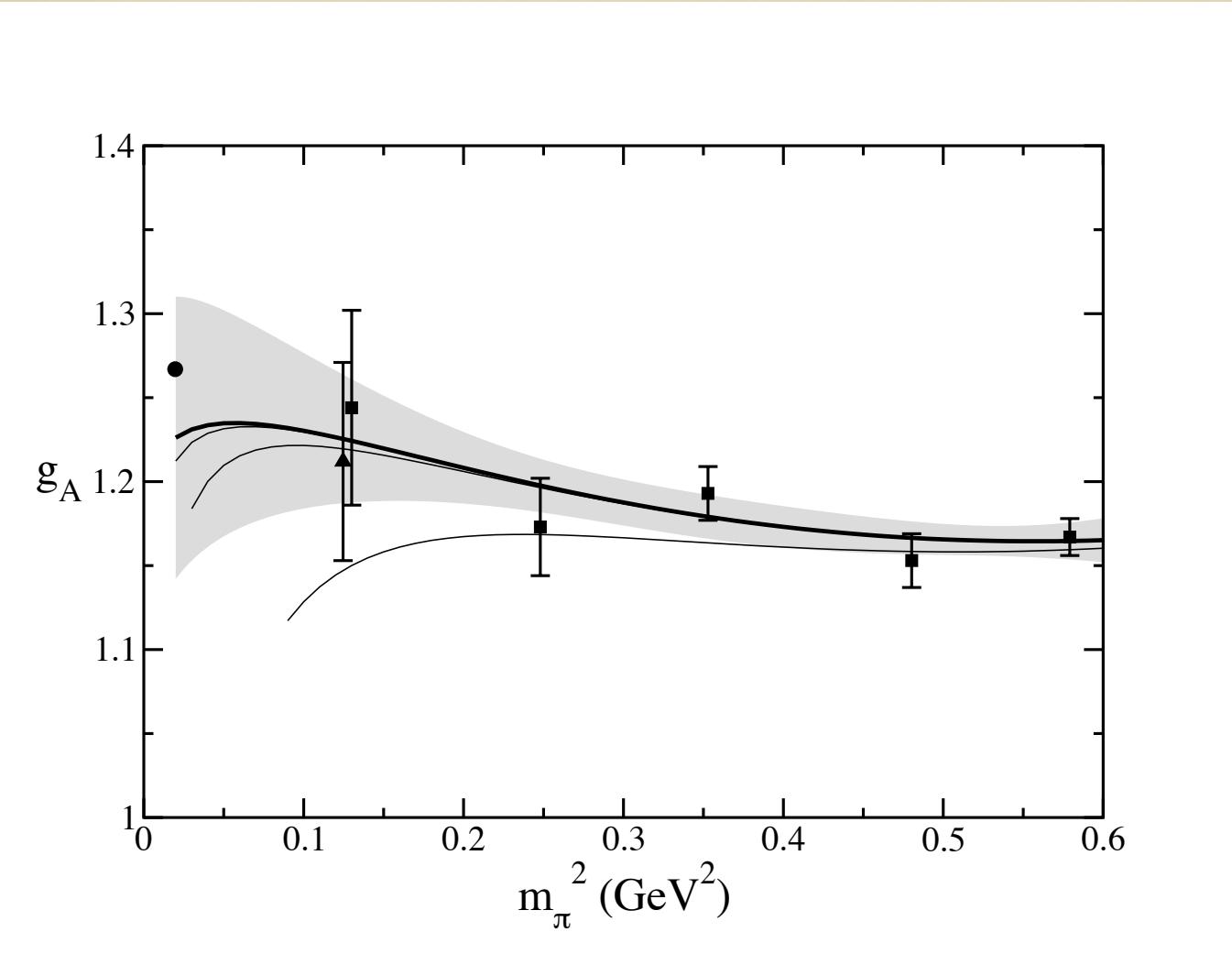
# Chiral expansion of axial charge

---

$$\begin{aligned}
 \Gamma_{NN} = g_A & - i \frac{4}{3f^2} [4g_A^3 J_1(m_\pi, 0, \mu) \\
 & + 4(g_{\Delta N}^2 g_A + \frac{25}{81} g_{\Delta N}^2 g_{\Delta\Delta}) J_1(m_\pi, \Delta, \mu) \\
 & + \frac{3}{2} g_A R_1(m_\pi, \mu) \\
 & - \frac{32}{9} g_{\Delta N} g_A N_1(m_\pi, \Delta, \mu)] \\
 & + C m_\pi^2
 \end{aligned}
 \quad \text{Beane and Savage hep-ph/0404131}$$

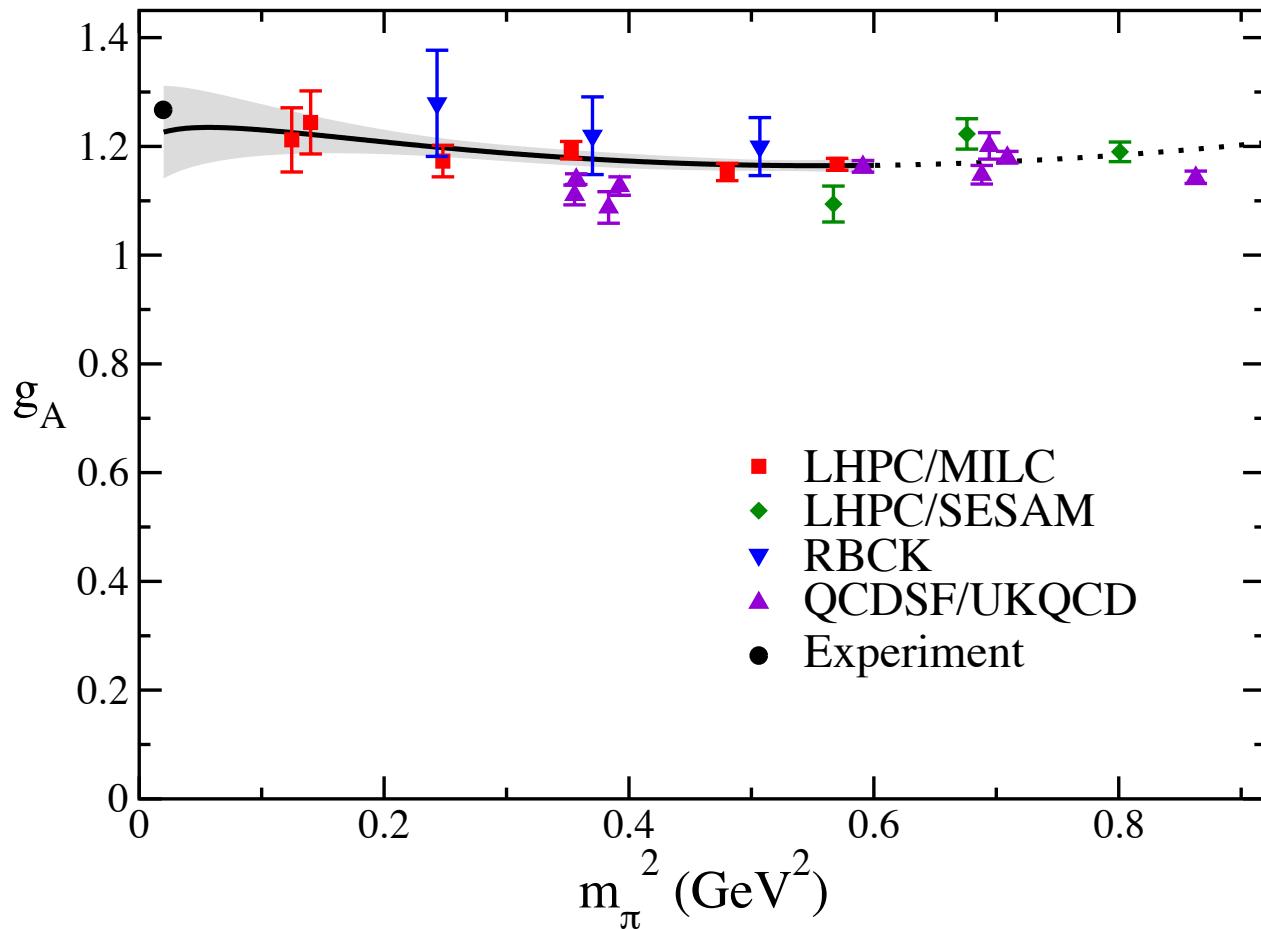
$$\begin{aligned}
 J_1(m, \Delta, \mu) &= -\frac{3}{4} \frac{i}{16\pi^2} \left[ (m^2 - 2\Delta^2) \log \frac{m^2}{\mu^2} + 2\Delta F(m, \Delta) \right] \\
 R_1(m, \mu) &= \frac{i}{16\pi^2} m^2 \left[ \Gamma(\epsilon) + 1 - \log \frac{m^2}{\mu^2} \right] \\
 N_1(m, \Delta, \mu) &= -\frac{3}{4} \frac{i}{16\pi^2} \left[ (m^2 - \frac{2}{3}\Delta^2) \log \frac{m^2}{\mu^2} + \frac{2}{3}\Delta F(m, \Delta) + \frac{2}{3} \frac{m^2}{\Delta} [\pi m - F(m, \Delta)] \right] \\
 f(m, \Delta) &= \sqrt{\Delta^2 - m^2 - i\epsilon} \log \left( \frac{\Delta - \sqrt{\Delta^2 - m^2 - i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 - i\epsilon}} \right)
 \end{aligned}$$

# Nucleon axial charge $g_A$    $\langle 1 \rangle_{\Delta q}^{u-d}$



LHPC hep-lat/0510062

# Nucleon axial charge $g_A$    $\langle 1 \rangle_{\Delta q}^{u-d}$



# Chiral Extrapolation of Moments

---

# Chiral extrapolation of GPD's

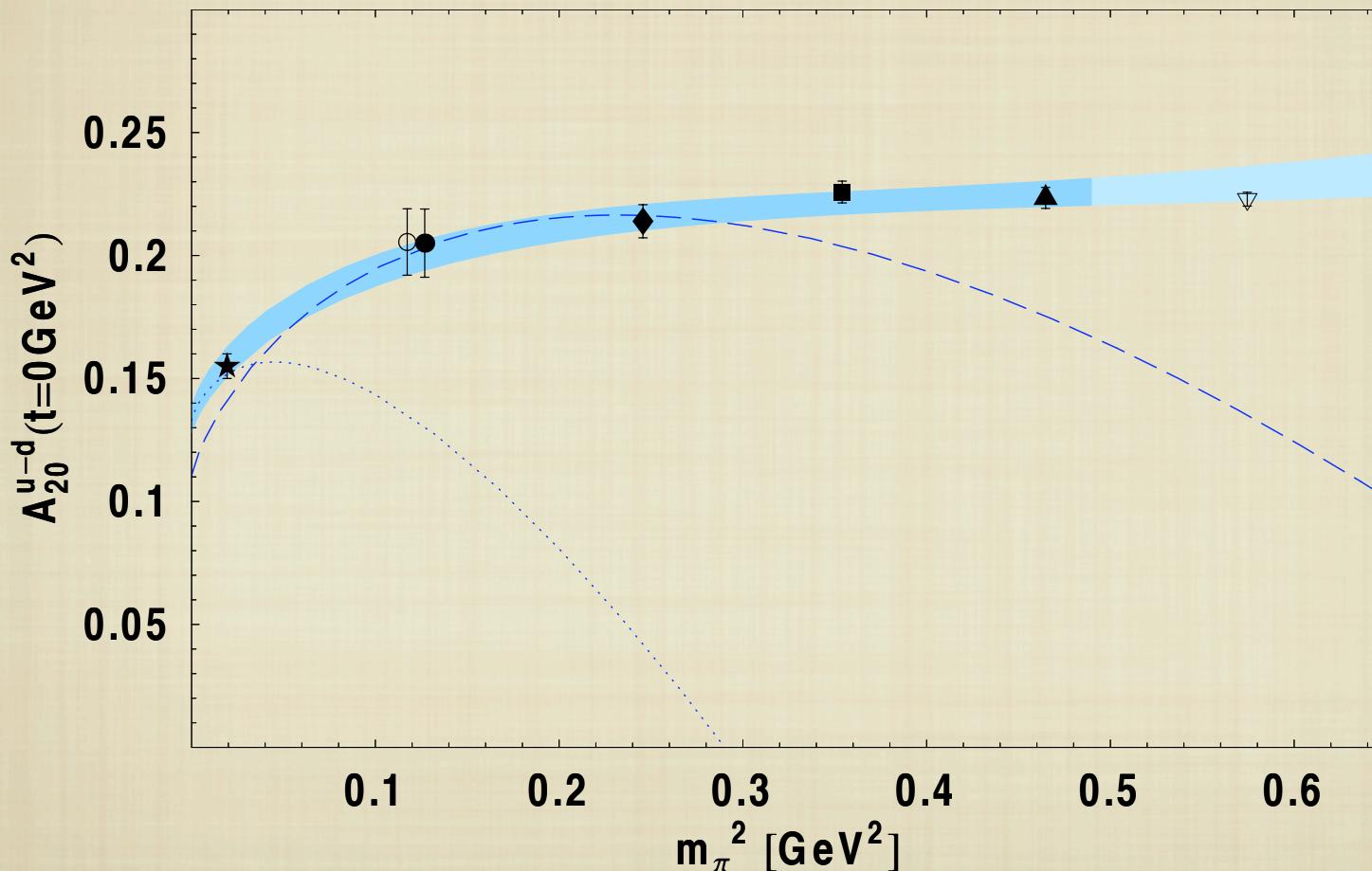
---

Haegler et al, LHPC, Phys Rev D77, 094502 (2008)

- Fundamental problem - large pion masses
- Covariant Baryon Chiral Perturbation theory gives consistent fit to matrix elements of twist-2 operators for wide range of masses  
(Dorati, Gail, Hemmert, Nucl Phys A798, 96 (2008))
- HBChPT expands in  $\epsilon = \left\{ \frac{m_\pi}{\Lambda_\chi}, \frac{p}{\Lambda_\chi}, \frac{m_\pi}{M_N^0}, \frac{p}{M_N^0} \right\}$   
 $\Lambda_\chi = 4\pi f_\pi \sim 1.17 \text{ GeV}, \quad M_N^0 \sim 890 \text{ MeV}$
- CBChPT resums all orders of  $\left( \frac{1}{M_N^0} \right)^m$

# Chiral extrapolation of $\langle x \rangle_q^{u-d} = A_{20}^{u-d}(t=0)$

Chiral extrapolation  $O(p^4)$  CBChPT (Dorati, Hemmert, et. al.)

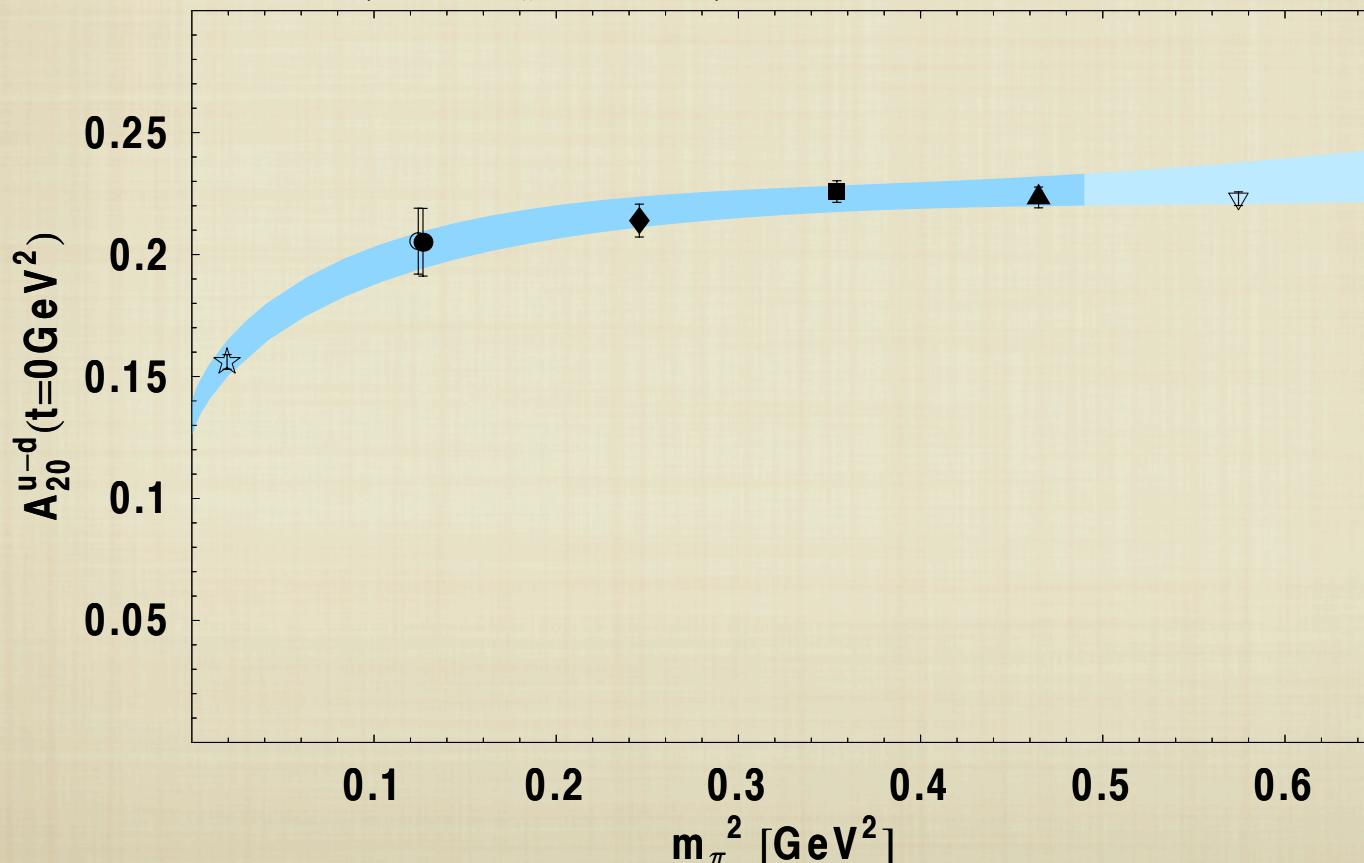


# Chiral extrapolation of $\langle x \rangle_q^{u-d} = A_{20}^{u-d}(t=0)$

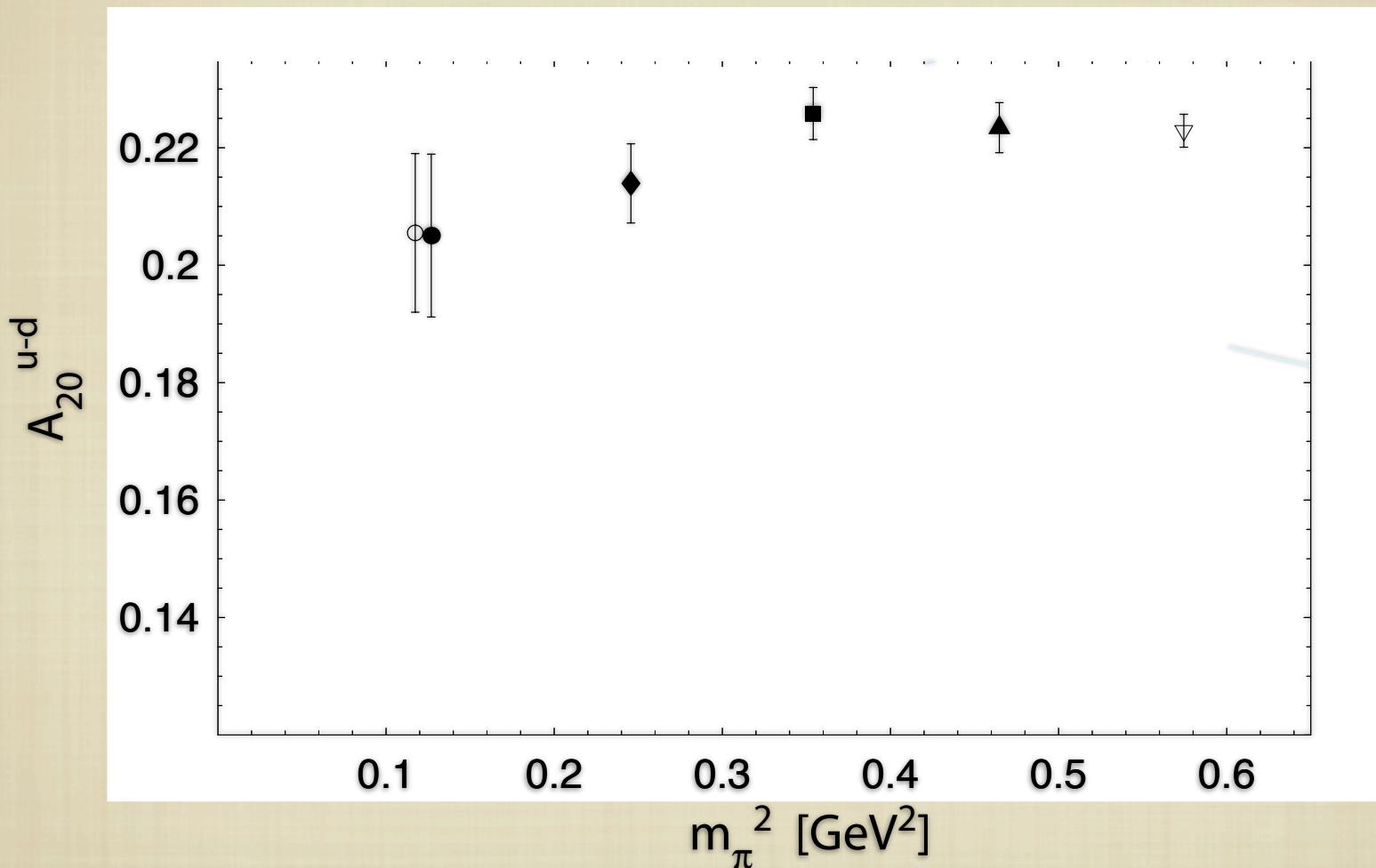
Chiral extrapolation  $O(p^2)$  CBChPT (Dorati, et al, NP A798, 96 (2008))

Global fit to A, B, C with 9 fit parameters

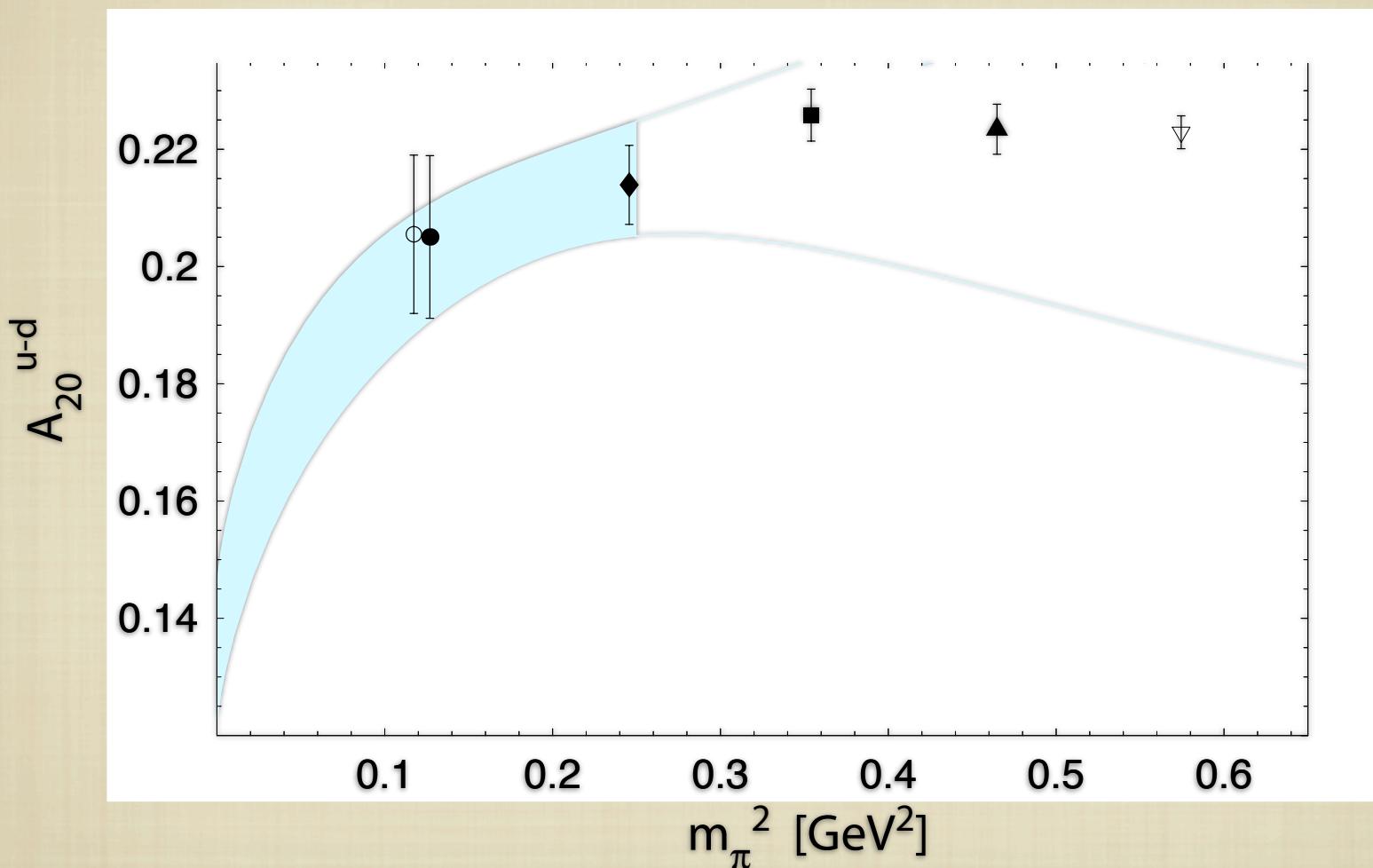
$$\begin{aligned} A_{20}^{u-d}(t, m_\pi) &= A_{20}^{0,u-d}(f_A(m_\pi) + \frac{g_A^2}{192\pi^2 f_\pi^2} h_A(t, m_\pi)) + \tilde{A}_{20}^{0,u-d} j_A(m_\pi) + A_{20}^{m_\pi,u-d} m_\pi^2 + A_{20}^t t \\ &\sim a \left( 1 - \frac{3g_A^2 + 1}{4\pi f_\pi^2} m_\pi^2 \ln m_\pi^2 \right) + b m_\pi^2 \dots \end{aligned}$$



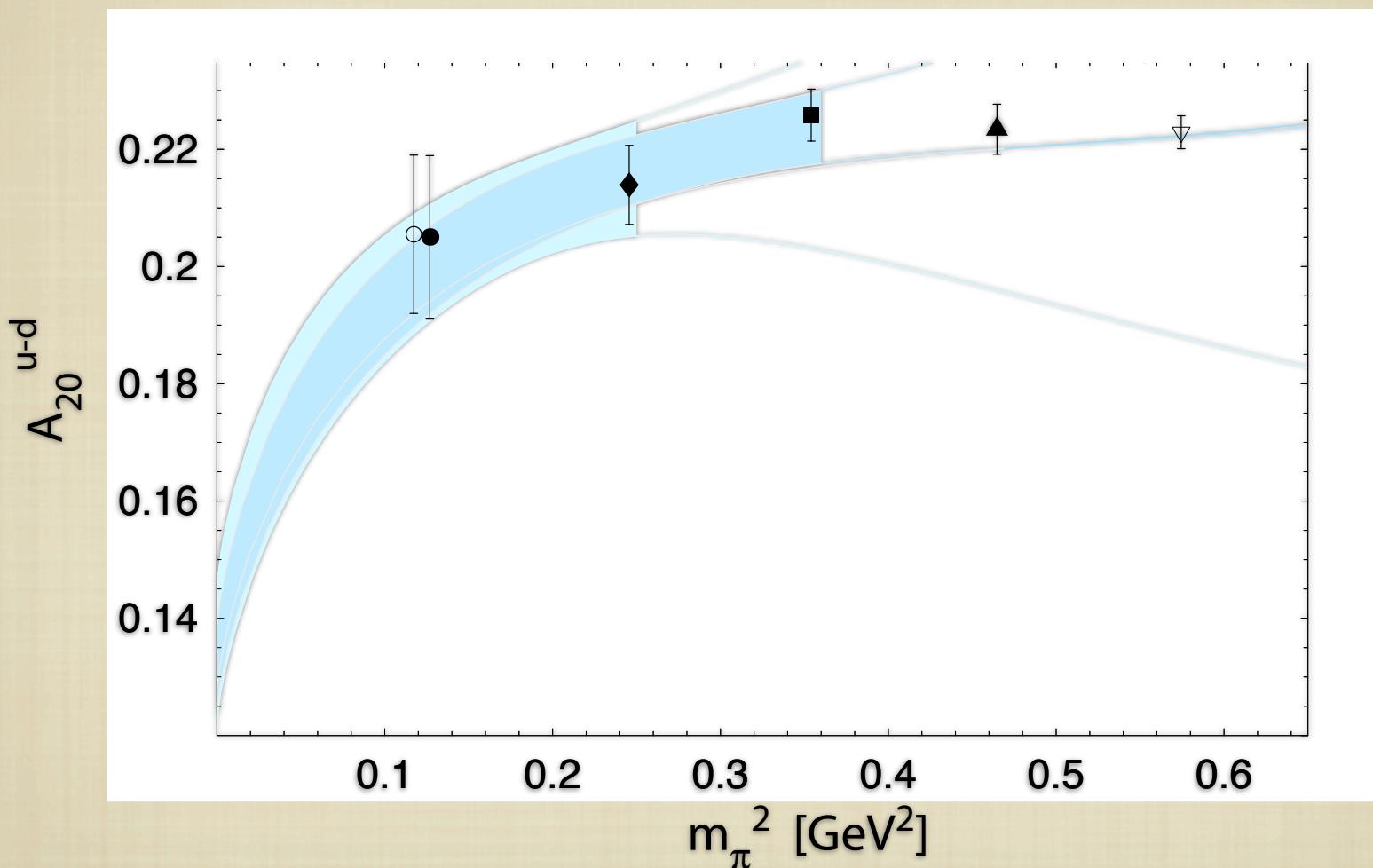
# Chiral extrapolation of $\langle x \rangle$



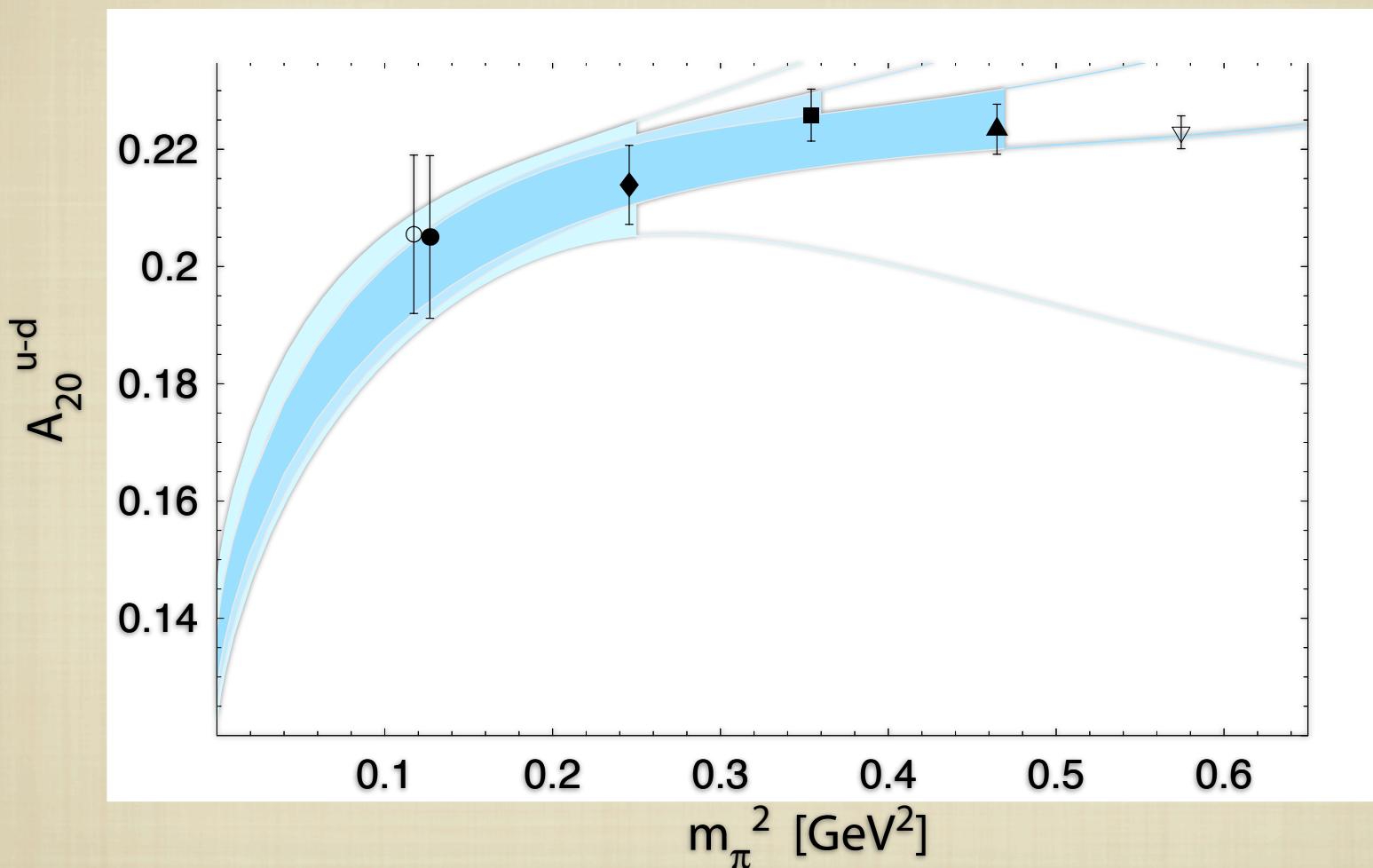
# Chiral extrapolation of $\langle x \rangle$



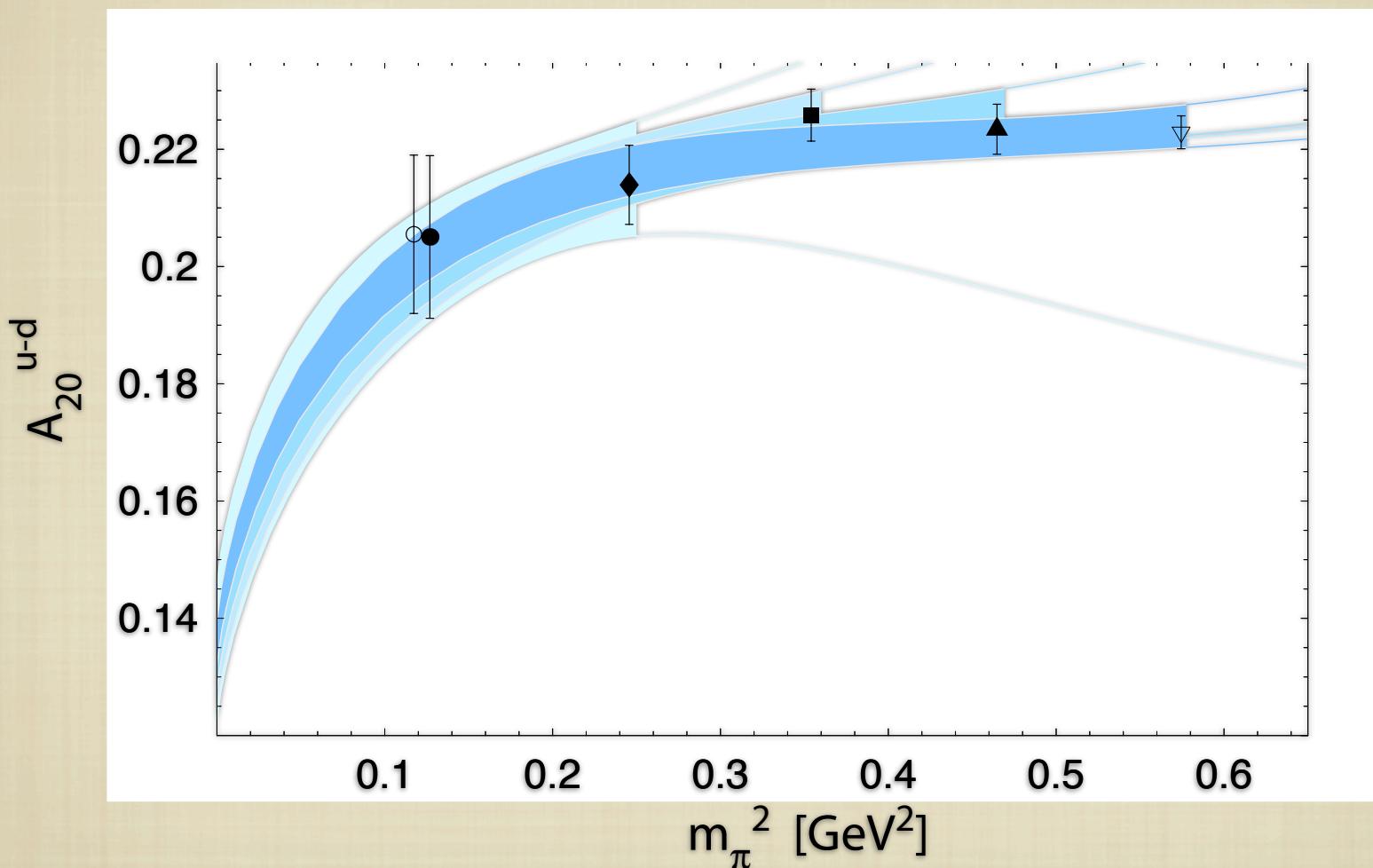
# Chiral extrapolation of $\langle x \rangle$



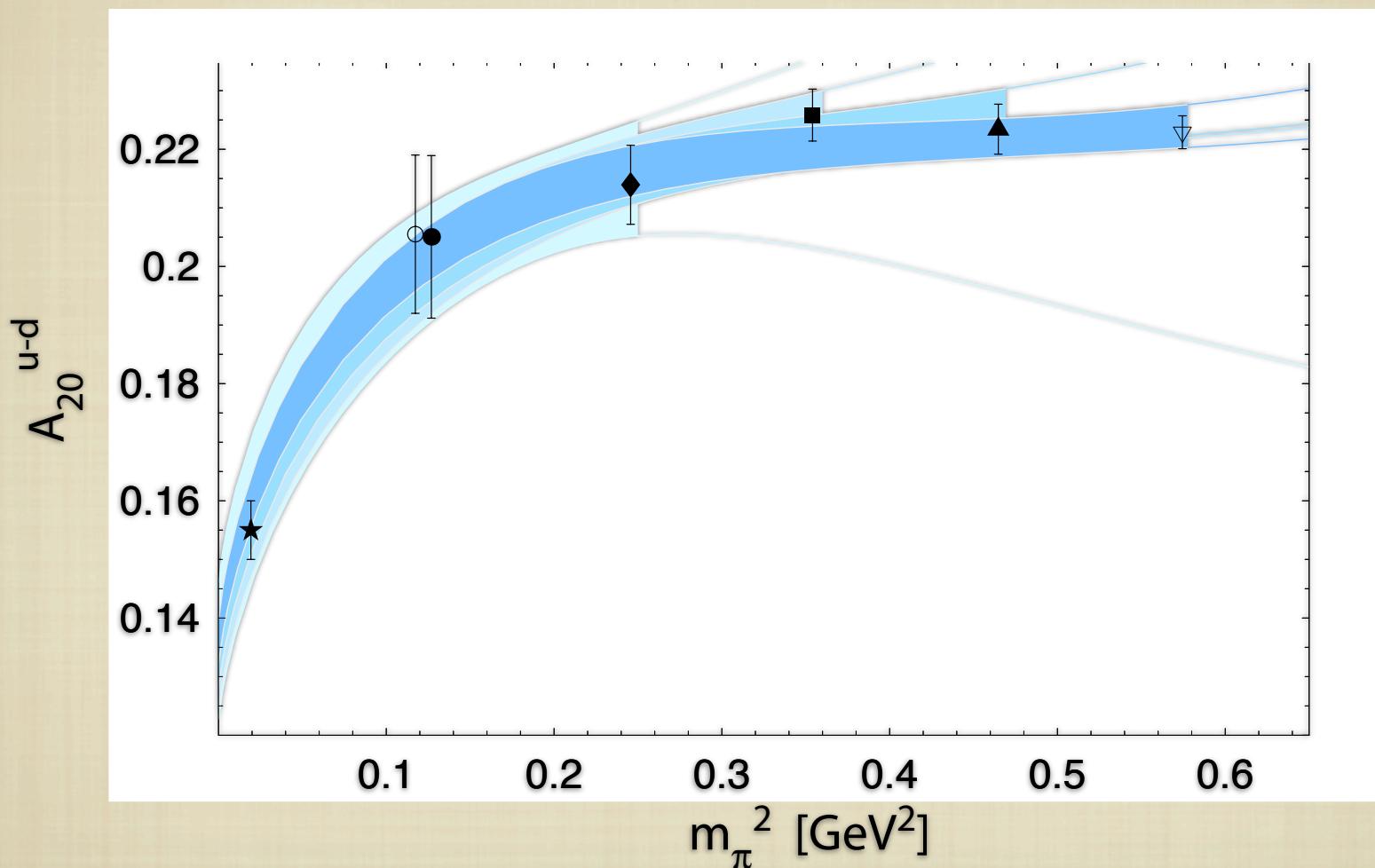
# Chiral extrapolation of $\langle x \rangle$



# Chiral extrapolation of $\langle x \rangle$



# Chiral extrapolation of $\langle x \rangle$



# Chiral Extrapolation of Moments

---

- for example, unpolarized moments

$$\langle x^n \rangle_{u-d} = a_n \left( 1 - \frac{(3g_{A,0}^2 + 1)}{(4\pi f_{\pi,0})^2} m_\pi^2 \ln \left( \frac{m_\pi^2}{\mu^2} \right) \right) + b'_n(\mu) m_\pi^2$$

- choose  $\mu = f_{\pi,0}$ , and at one loop  $g_{A,0} \rightarrow g_{A,m_\pi}$  and  $f_{\pi,0} \rightarrow f_{\pi,m_\pi}$

$$\langle x^n \rangle_{u-d} = a_n \left( 1 - \frac{(3g_{A,m_\pi}^2 + 1)}{(4\pi)^2} \frac{m_\pi^2}{f_{\pi,m_\pi}^2} \ln \left( \frac{m_\pi^2}{f_{\pi,m_\pi}^2} \right) \right) + b_n \frac{m_\pi^2}{f_{\pi,m_\pi}^2}$$

- self consistently  $g_A \rightarrow g_{A,\text{lat}}$ ,  $f_\pi \rightarrow f_{\pi,\text{lat}}$ ,  $m_\pi \rightarrow m_{\pi,\text{lat}}$

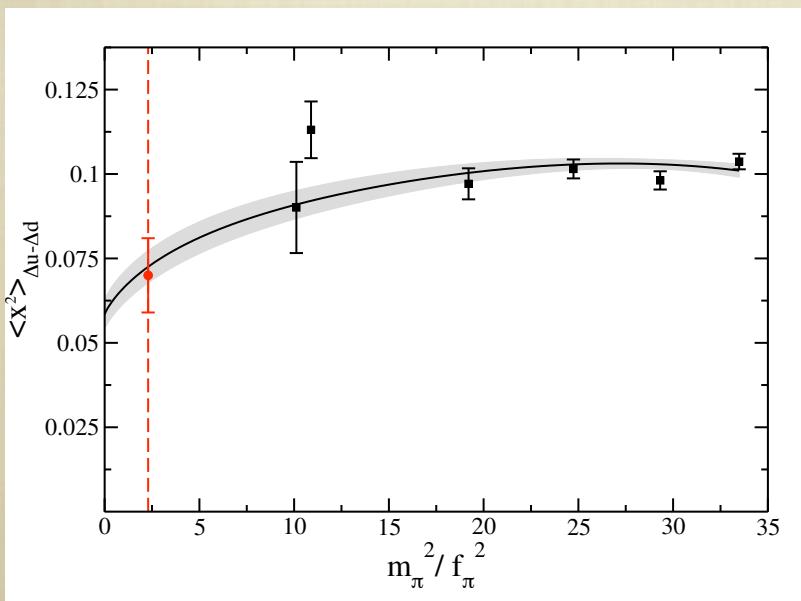
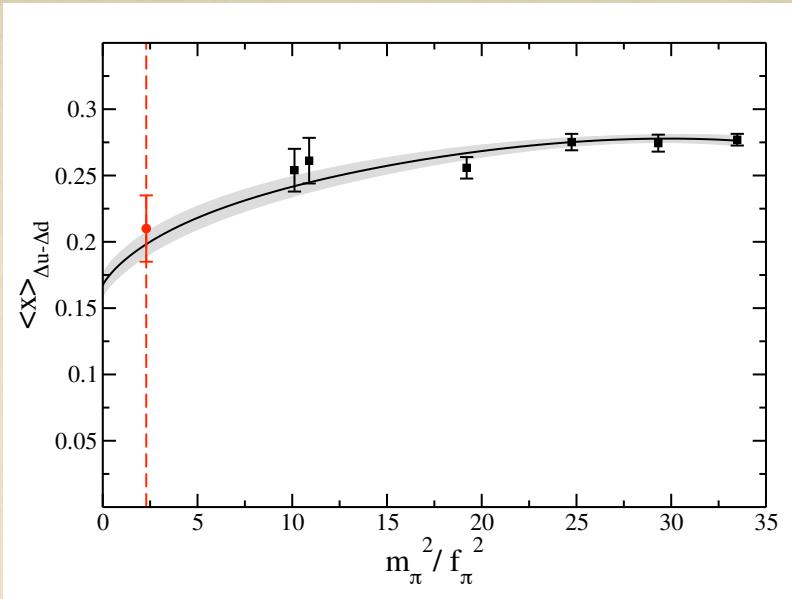
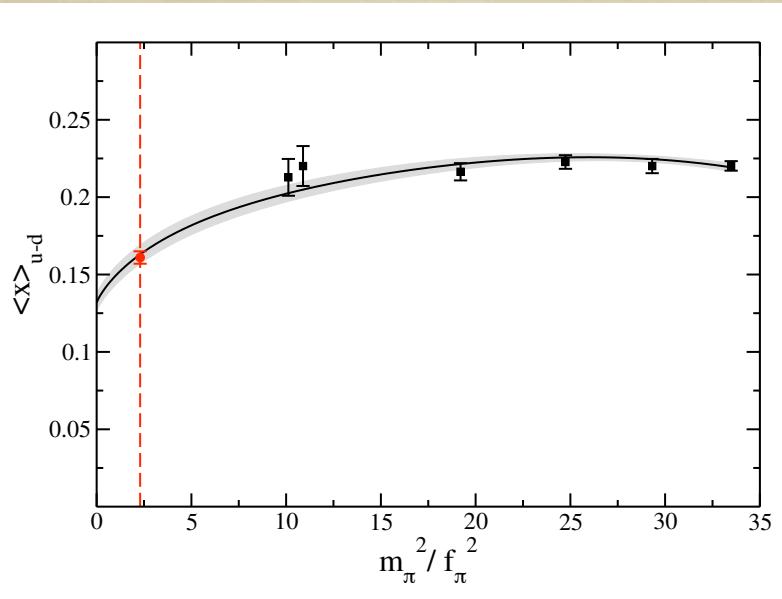
$$\langle x^n \rangle_{u-d} = a_n \left( 1 - \frac{(3g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln \left( \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \right) \right) + b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

- similarly for the helicity and transversity moments

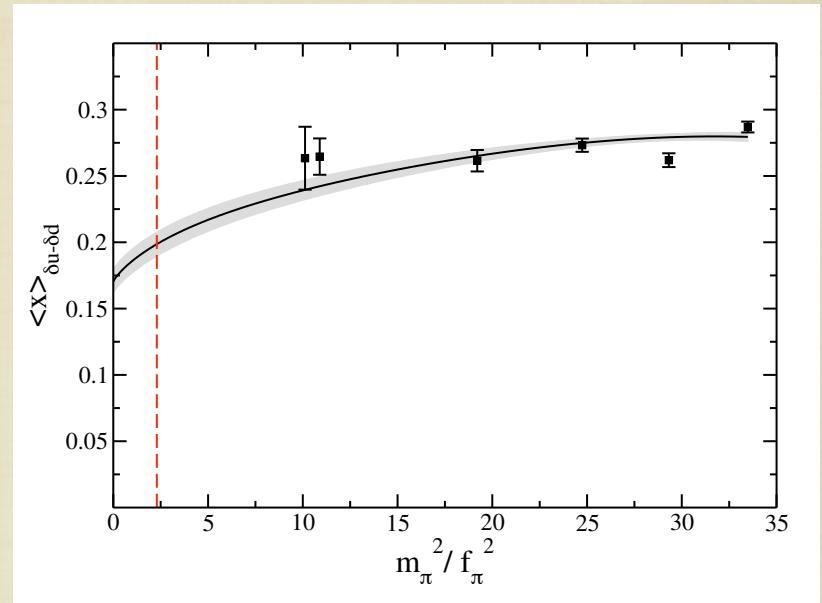
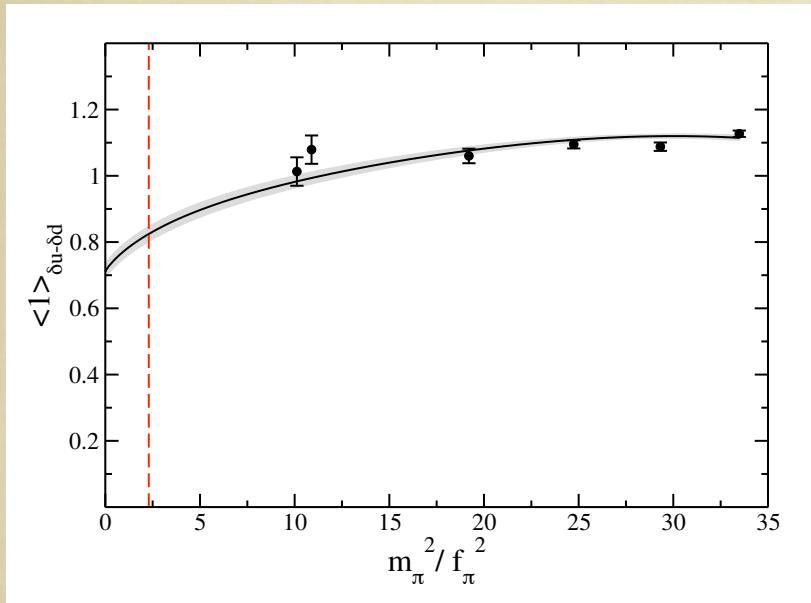
$$\langle x^n \rangle_{\Delta u - \Delta d} = \Delta a_n \left( 1 - \frac{(2g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln \left( \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \right) \right) + \Delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

$$\langle x^n \rangle_{\delta u - \delta d} = \delta a_n \left( 1 - \frac{(4g_{A,\text{lat}}^2 + 1)}{2(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln \left( \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \right) \right) + \delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

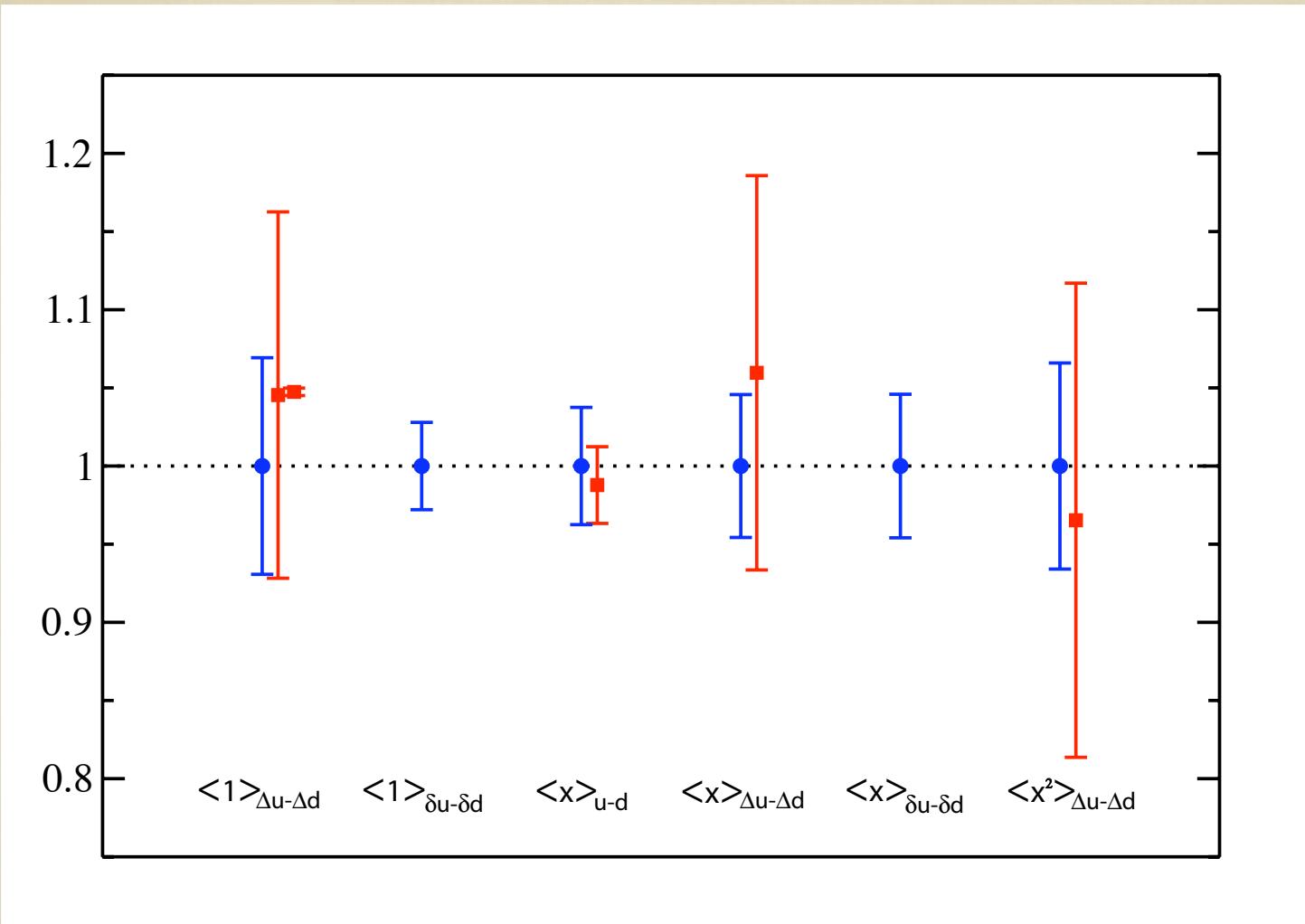
# Chiral Extrapolation of Moments



# Chiral Extrapolation of Moments

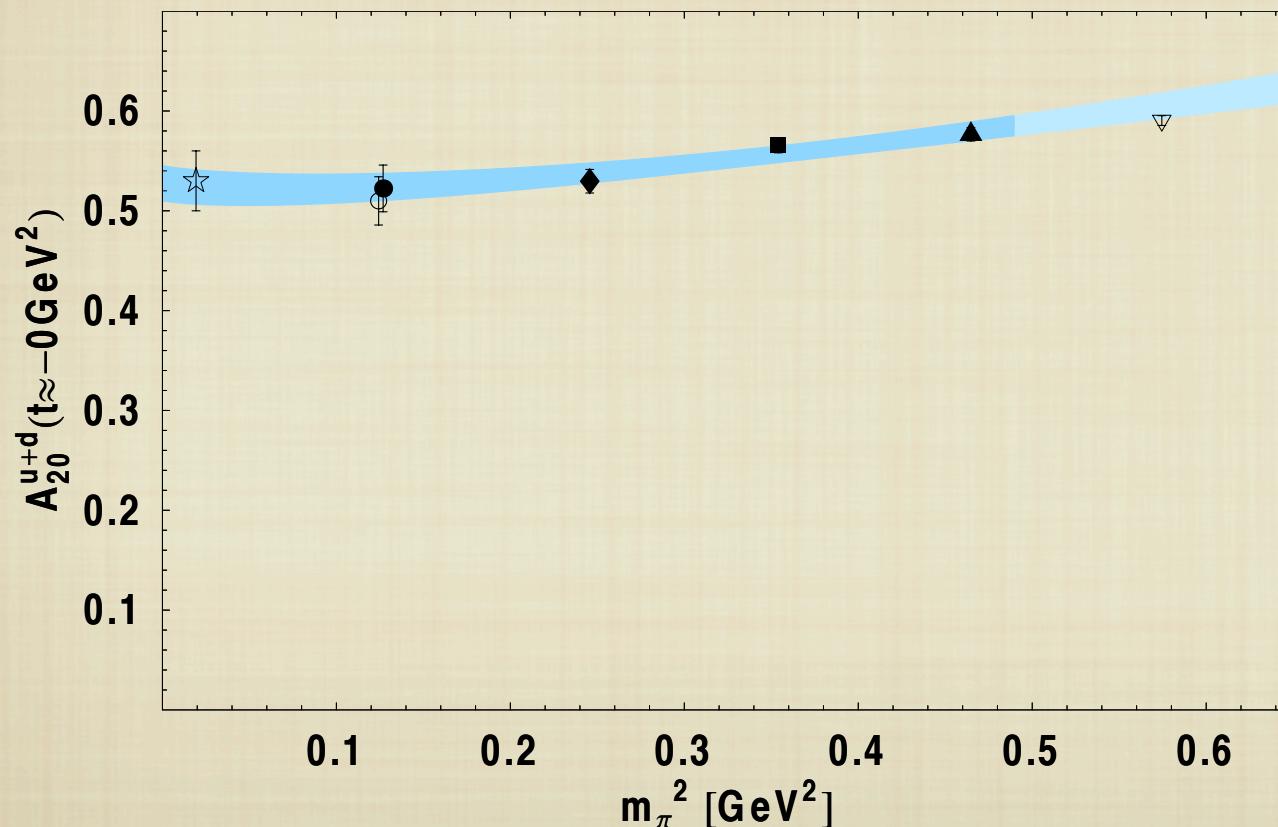


# Chiral Extrapolation of Moments



# Chiral extrapolation of $\langle x \rangle_q^{u+d} = A_{20}^{u+d}(t = 0)$

Chiral extrapolation  $O(p^2)$  CBChPT (Dorati, Hemmert, et. al.)  
Note: connected diagrams only



# Form Factors

---

# Electromagnetic form factors

---

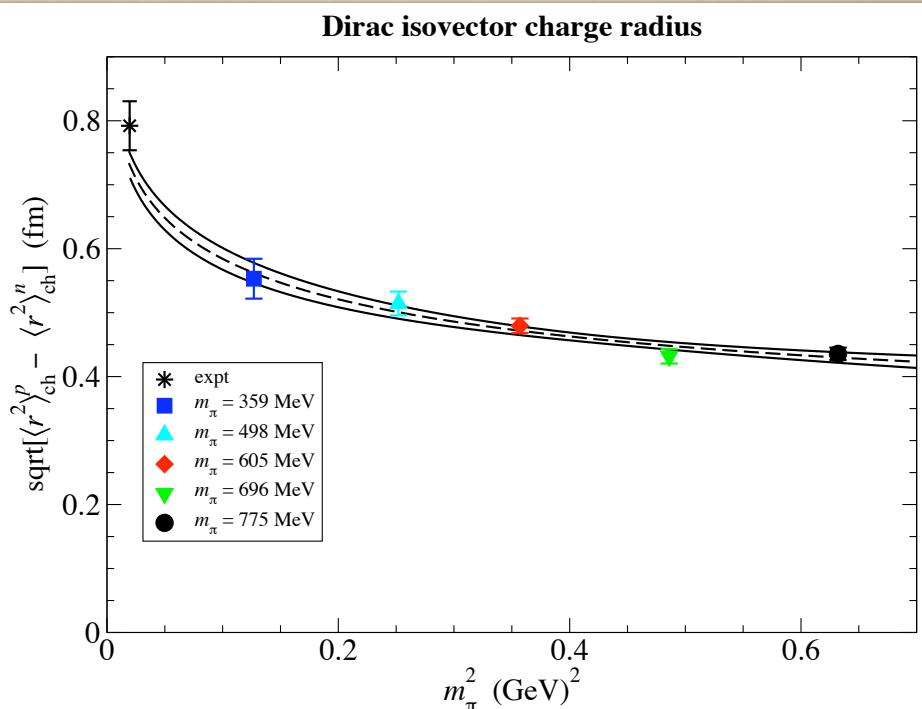
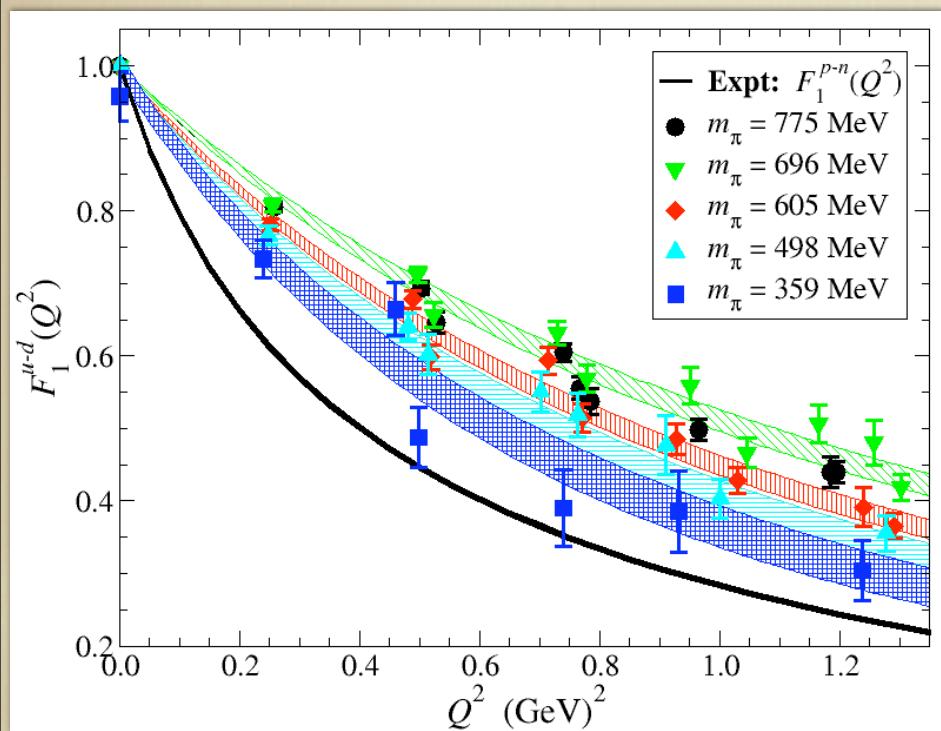
- Simplest off-diagonal matrix element

$$\langle p | \bar{\psi} \gamma^\mu \psi | p' \rangle = \bar{u}(p) [F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m}] u(p')$$

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2) \quad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

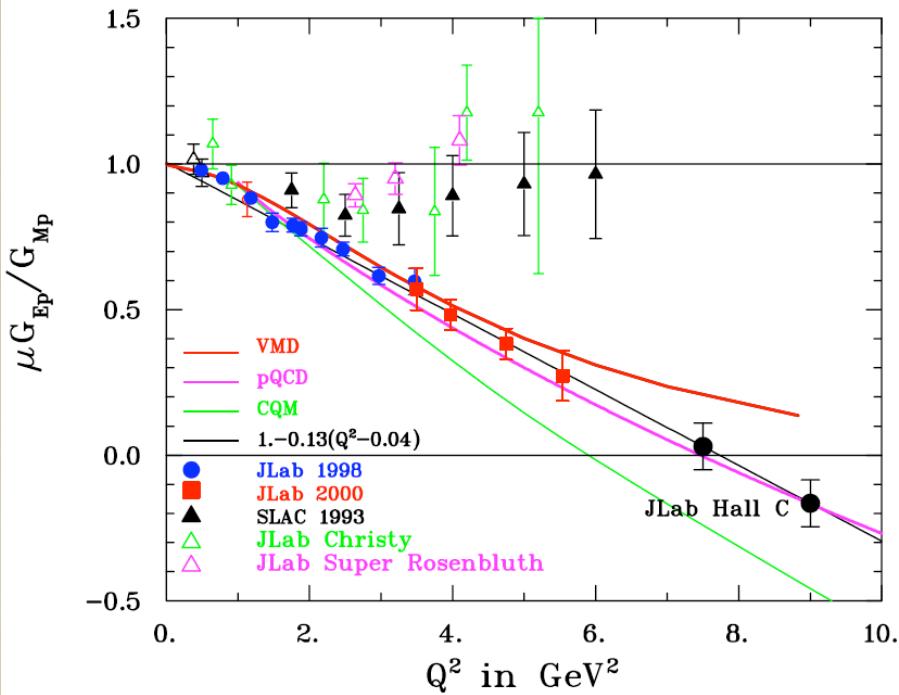
- Fourier transform of charge density if  $L_{\text{system}} \gg L_{\text{wavepacket}} \gg \frac{1}{m}$ 
  - Pb:  $5 \text{ fm} \gg 10^{-5} \text{ fm}$ , Proton:  $0.8 \text{ fm} \sim 0.2 \text{ fm}$ : marginal
  - For transverse Fourier transform of light cone w. f.,  $m \rightarrow p_+ \sim \infty$
- Large  $q^2$ : ability of one quark to share  $q^2$  with other constituents to remain in ground state -  $q^2$  counting rules

# $F_1$ Isovector Form Factor

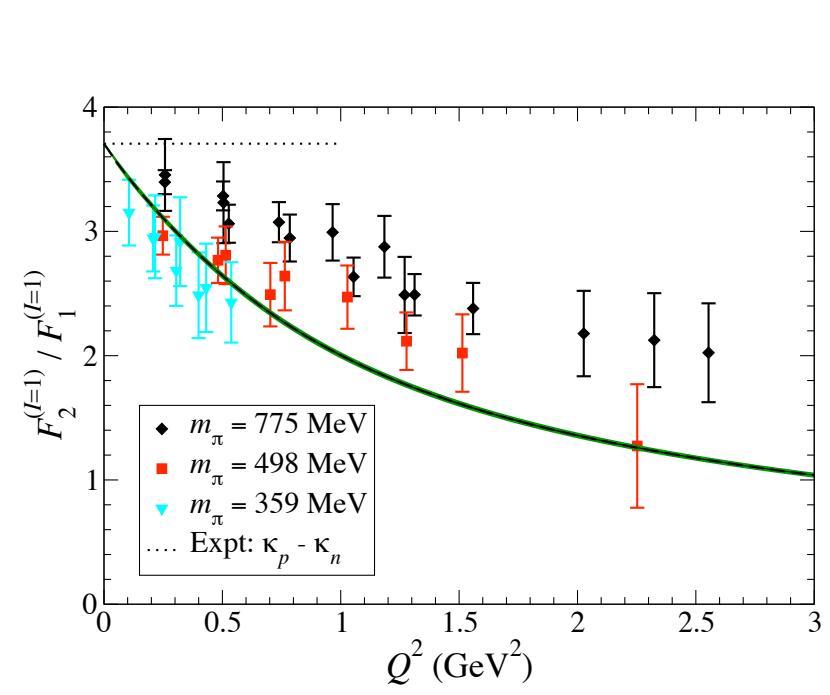


$$\langle r^2 \rangle^{u-d} = a_0 - \frac{(1 + 5g_A^2)}{(4\pi f_\pi)^2} \log \left( \frac{m_\pi^2}{m_\pi^2 + \Lambda^2} \right)$$

# Form factor ratio: $F_2 / F_1$



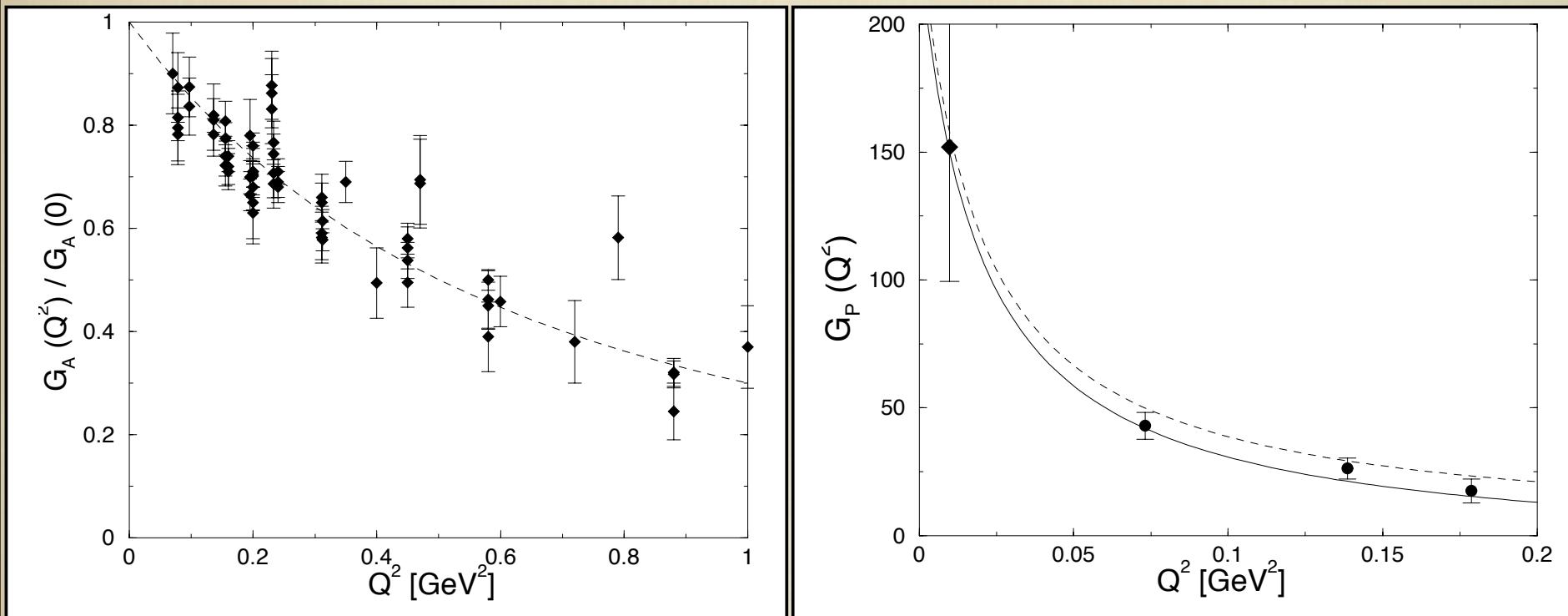
Polarization transfer at JLab



Lattice results

# Polarized Nucleon Form Factors $G_A$ and $G_P$

$$\langle p | \bar{\psi} \gamma^\mu \gamma_5 \psi | p' \rangle = \bar{u}(p) [G_A(q^2) \gamma^\mu \gamma_5 + q^\mu \gamma_5 G_P(q^2) + \sigma^{\mu\nu} \gamma_5 q_\nu G_M(q^2)] u(p')$$

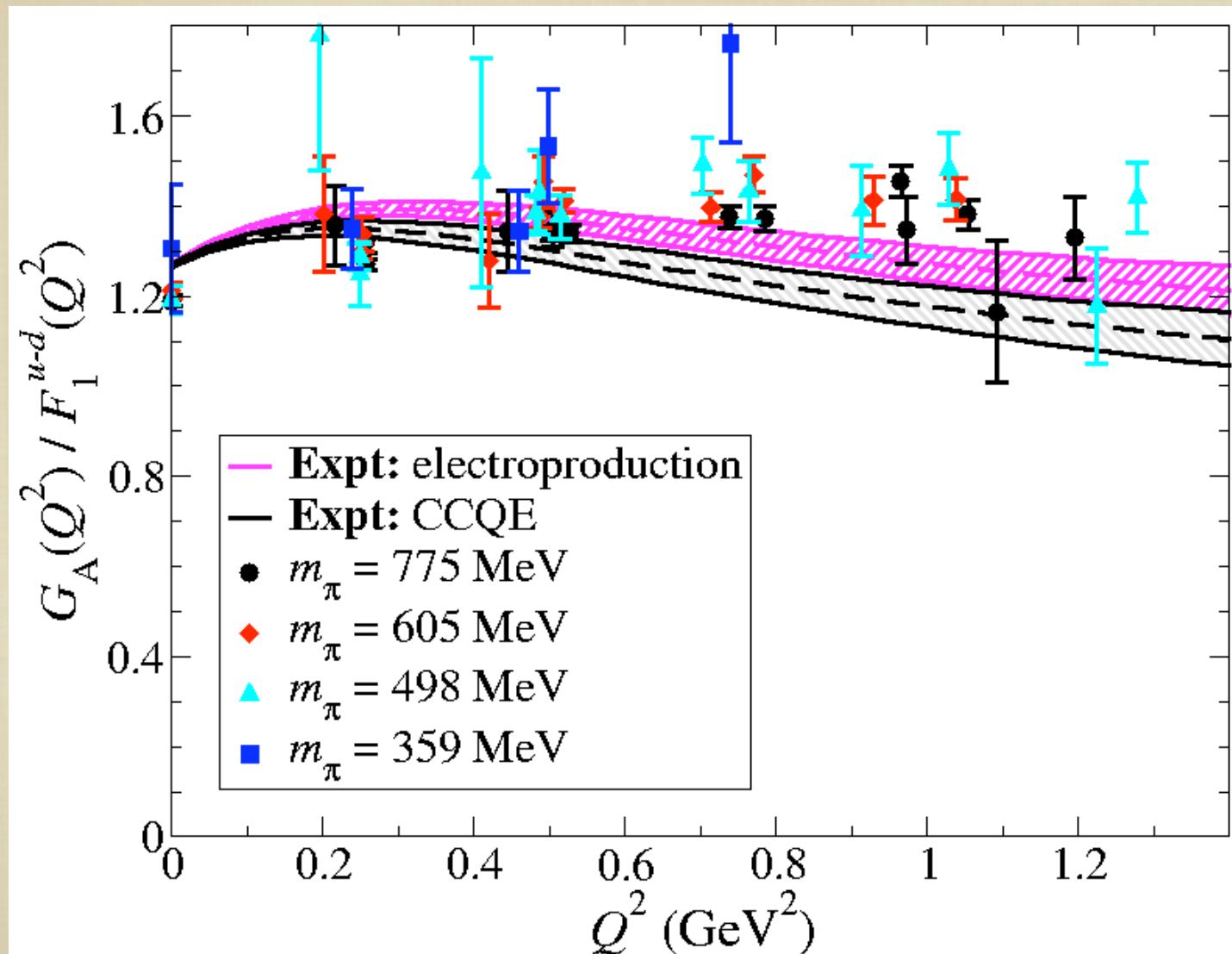


Bernard, Elouadrhiri, Meissner, J. Phys. G Nucl. Part. Phys. 2002, R1

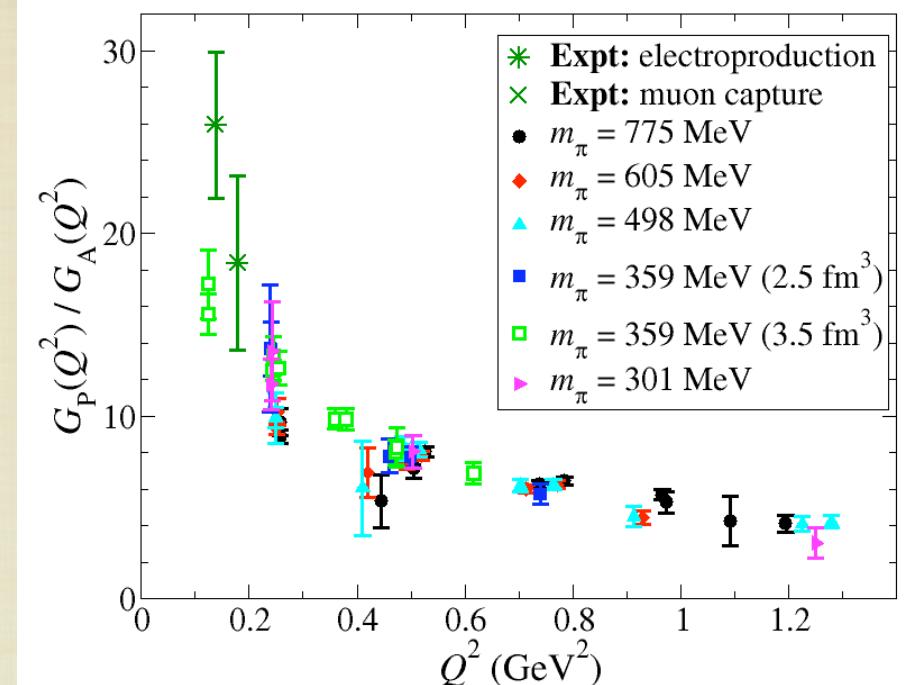
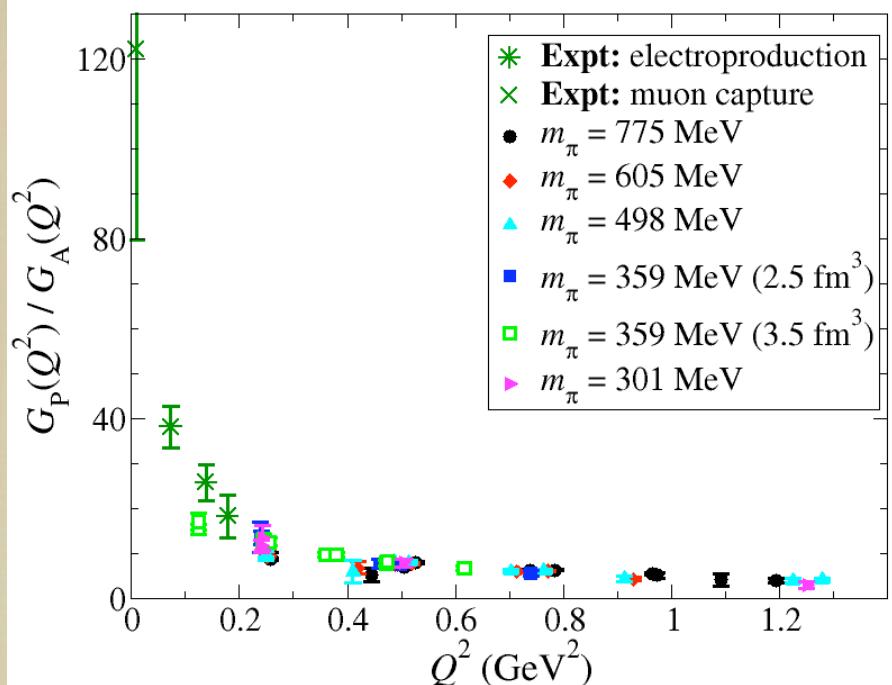
pion electroproduction ◆  
 $\nu_\mu n \rightarrow \mu^- p$

pion electroproduction ●  
 $\mu^- p \rightarrow \nu_\mu n$  ◆

# Form factor ratio: $G_A/F_1$



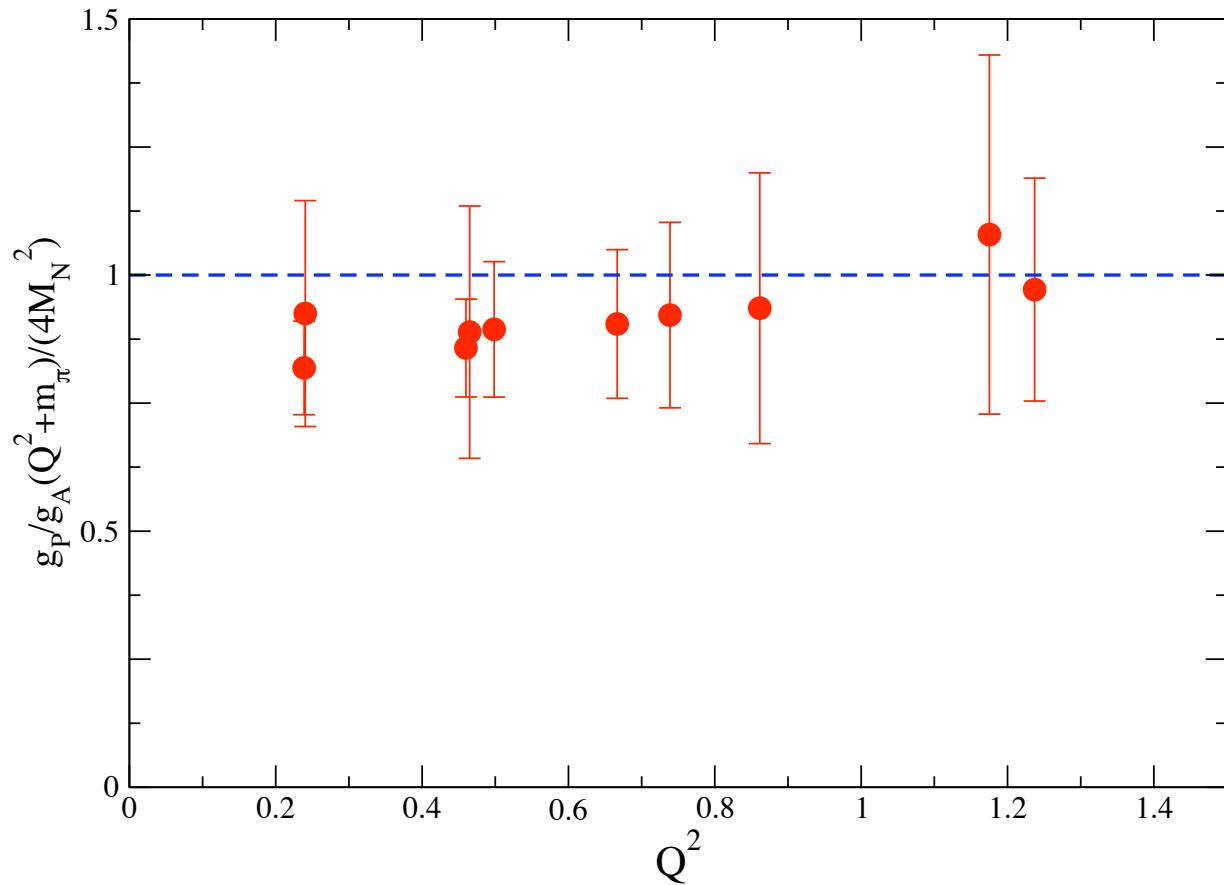
# Form factor ratio: $G_P/G_A$



soft pion pole:

$$G_P(q^2) \sim \frac{4M^2 G_A(q^2)}{q^2 - m_\pi^2}$$

# Form factor ratio: $G_P/G_A$



# Generalized Form Factors

---

# Generalized Parton Distributions

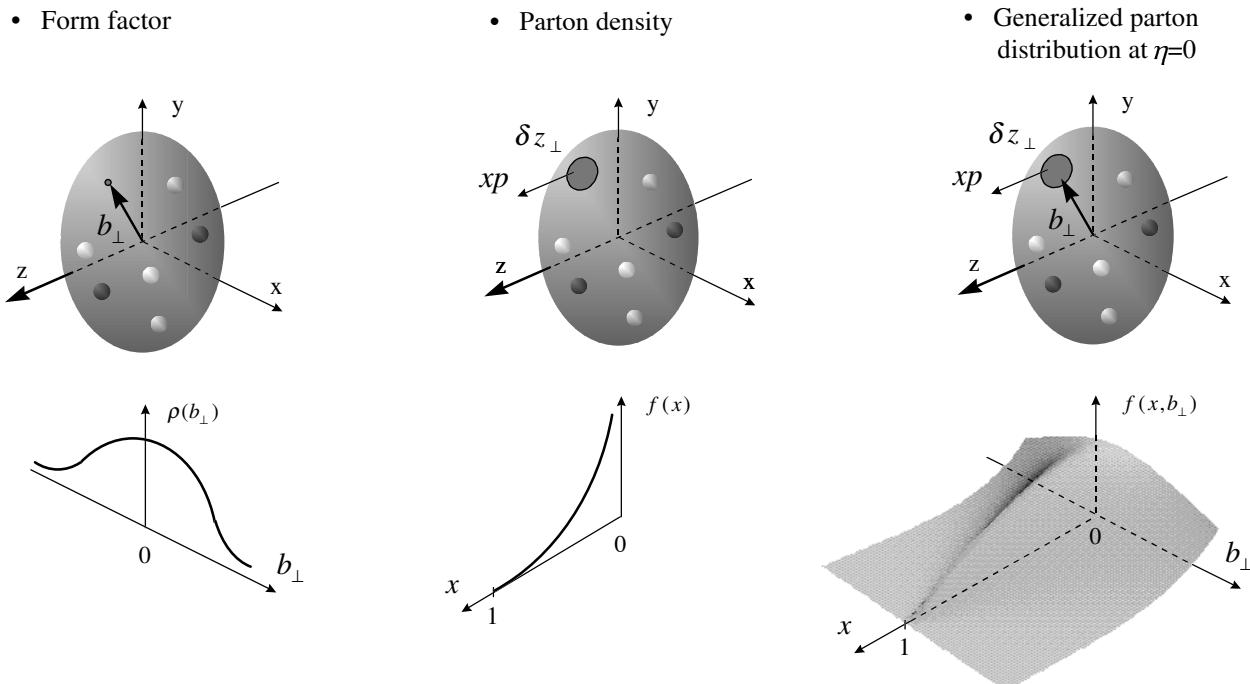


Fig. from G. Schierholz

# Generalized form factors

---

$$\mathcal{O}_q^{\{\mu_1 \mu_2 \dots \mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} \psi_q \quad \bar{P} = \frac{1}{2}(P' + P)$$

$$\begin{aligned} \langle P' | \mathcal{O}^{\mu_1} | P \rangle &= \langle\langle \gamma^{\mu_1} \rangle\rangle A_{10}(t) \\ &+ \frac{i}{2m} \langle\langle \sigma^{\mu_1 \alpha} \rangle\rangle \Delta_\alpha B_{10}(t), \end{aligned} \quad \Delta = P' - P \quad t = \Delta^2$$

$$\begin{aligned} \langle P' | \mathcal{O}^{\{\mu_1 \mu_2\}} | P \rangle &= \bar{P}^{\{\mu_1} \langle\langle \gamma^{\mu_2}\} \rangle\rangle A_{20}(t) \\ &+ \frac{i}{2m} \bar{P}^{\{\mu_1} \langle\langle \sigma^{\mu_2}\}^\alpha \rangle\rangle \Delta_\alpha B_{20}(t) \\ &+ \frac{1}{m} \Delta^{\{\mu_1} \Delta^{\mu_2\}} C_2(t), \end{aligned}$$

$$\begin{aligned} \langle P' | \mathcal{O}^{\{\mu_1 \mu_2 \mu_3\}} | P \rangle &= \bar{P}^{\{\mu_1} \bar{P}^{\mu_2} \langle\langle \gamma^{\mu_3}\} \rangle\rangle A_{30}(t) \\ &+ \frac{i}{2m} \bar{P}^{\{\mu_1} \bar{P}^{\mu_2} \langle\langle \sigma^{\mu_3}\}^\alpha \rangle\rangle \Delta_\alpha B_{30}(t) \\ &+ \Delta^{\{\mu_1} \Delta^{\mu_2} \langle\langle \gamma^{\mu_3}\} \rangle\rangle A_{32}(t) \\ &+ \frac{i}{2m} \Delta^{\{\mu_1} \Delta^{\mu_2} \langle\langle \sigma^{\mu_3}\}^\alpha \rangle\rangle \Delta_\alpha B_{32}(t), \end{aligned}$$

# Limits of generalized form factors

---

- Moments of parton distributions  $t \rightarrow 0$

$$A_{n0} = \int dx x^{n-1} q(x)$$

- Electromagnetic form factors

$$A_{10} = F_1(t), \quad B_{10} = F_2(t)$$

- Total quark angular momentum

$$J_q = \frac{1}{2}[A(0)_{20} + B(0)_{20}]$$

# Sum Rules

---

## □ Momentum sum rule

$$1 = A_{20,q}(0) + A_{20,g}(0) = \langle x \rangle_q + \langle x \rangle_g$$

## □ Nucleon spin sum rule

$$\begin{aligned} \frac{1}{2} &= \frac{1}{2}(A_{20,q}(0) + A_{20,g}(0) + B_{20,q}(0) + B_{20,g}(0)) \\ &= \frac{1}{2}\Delta\Sigma_q + L_q + J_g \end{aligned}$$

## □ Vanishing of anomalous gravitomagnetic moment

$$0 = B_{20,q}(0) + B_{20,g}(0)$$

# Transverse Structure of Nucleon

---

# Transverse structure of nucleon

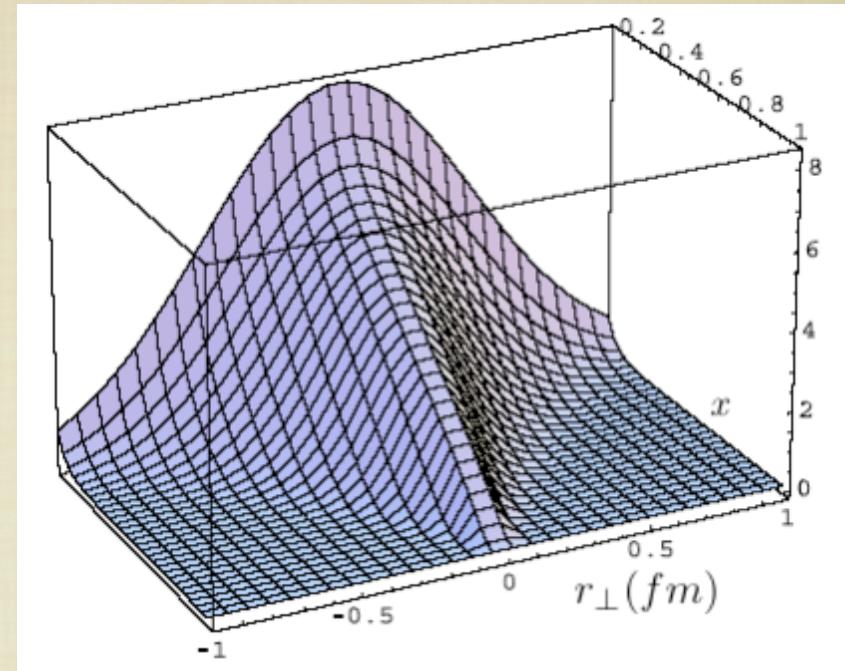
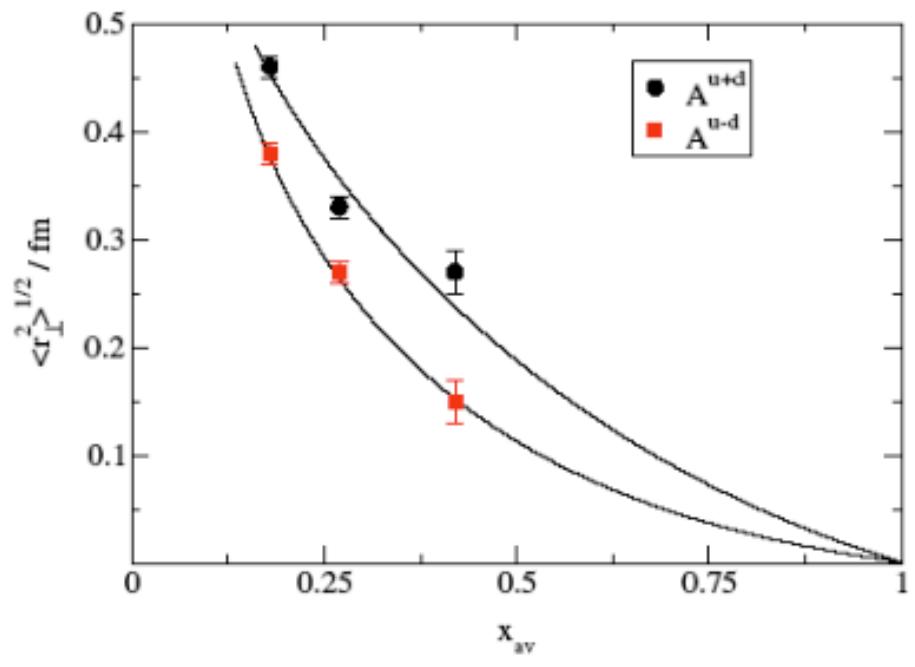
---

$H(x, 0, -\Delta_{\perp}^2)$  is transverse Fourier transform of light cone  
quark distribution  $q(x, r_{\perp})$  at momentum fraction  $x$

$$q(x, r_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} H(x, 0, -\Delta_{\perp}^2) e^{-ir_{\perp} \Delta_{\perp}}$$
$$\int dx x^{n-1} q(x, r_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} A(-\Delta_{\perp}^2) e^{-ir_{\perp} \Delta_{\perp}}$$

- $x \rightarrow 1$ : Single Fock space component  $\Rightarrow$  slope  $\rightarrow 0$
- $x \neq 1$ : Transverse structure  $\Rightarrow$  slope steeper

# Transverse size of light-cone wave function

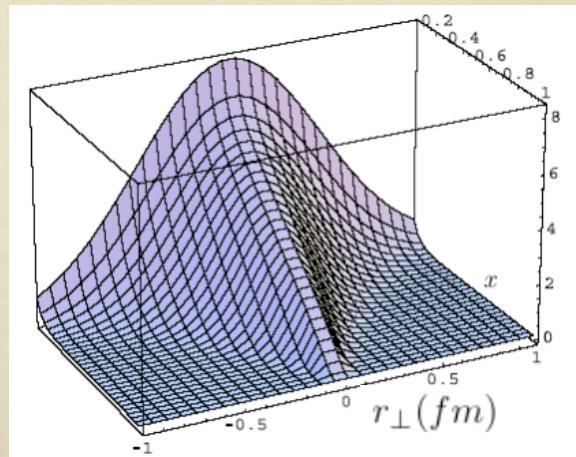
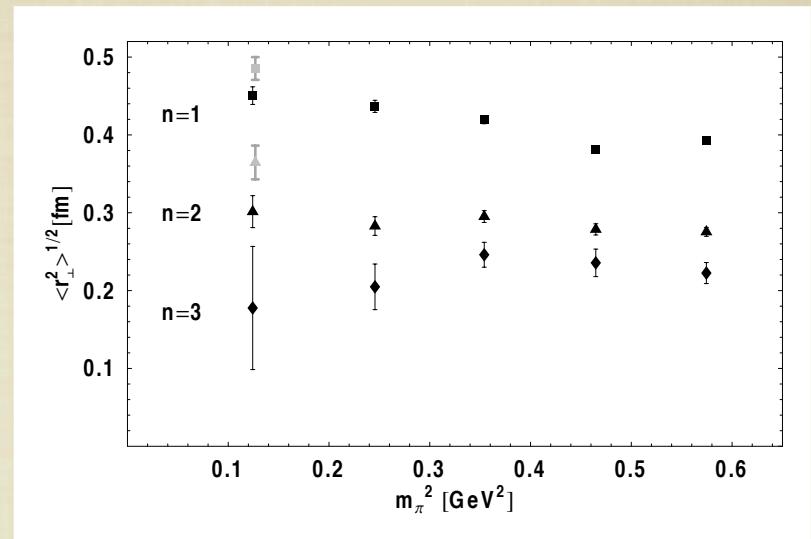
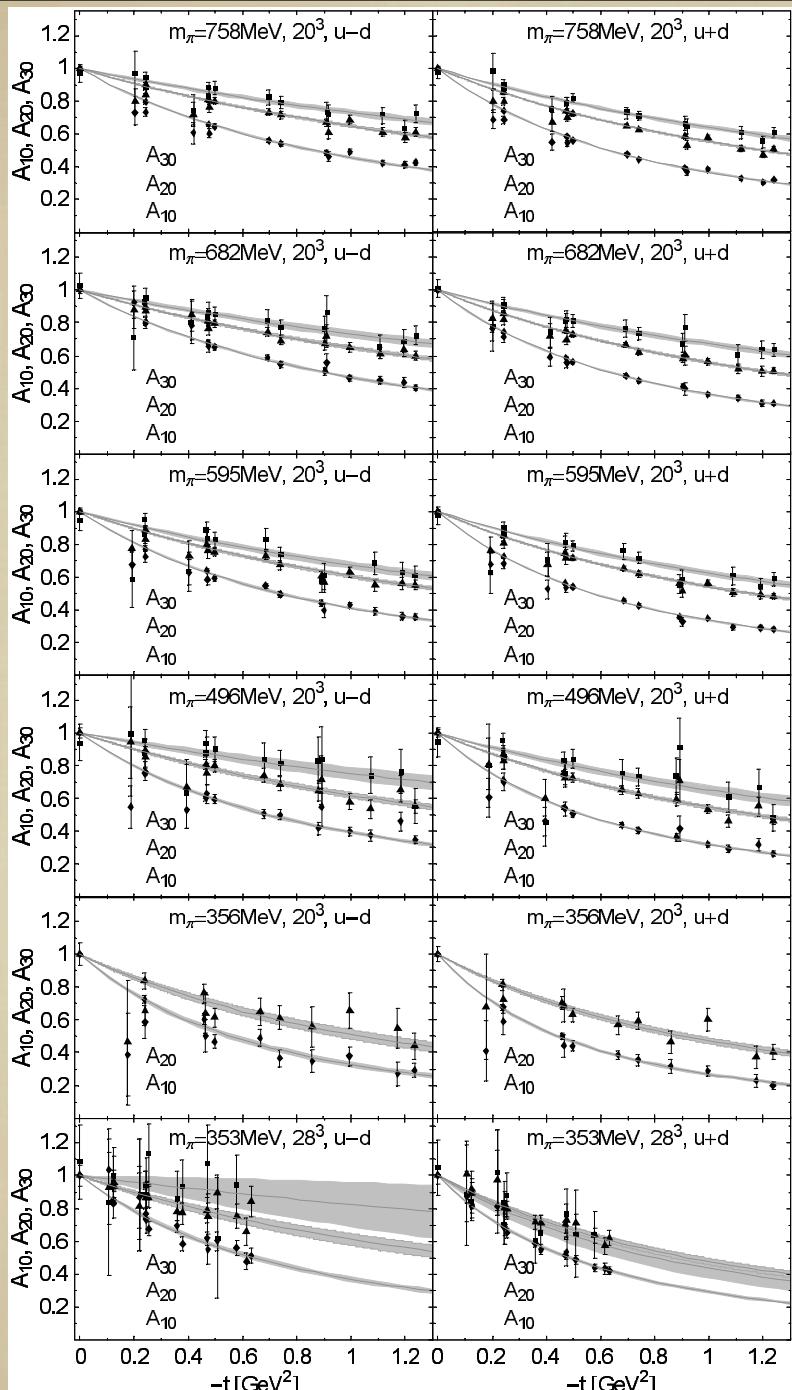


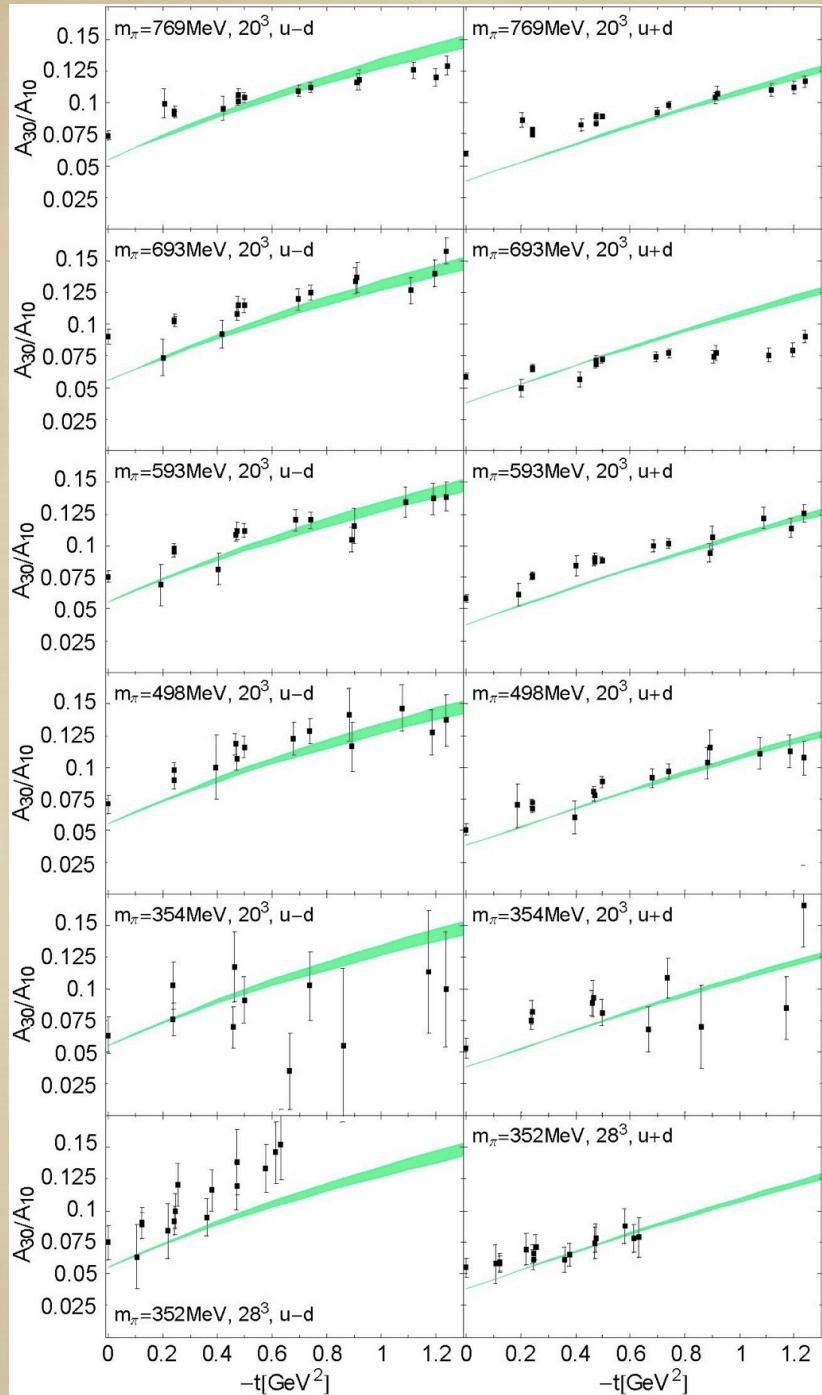
$$x_{\text{av}}^n = \frac{\int d^2 r_\perp \int dx x \cdot x^{n-1} q(x, \vec{r}_\perp)}{\int d^2 r_\perp \int dx x^{n-1} q(x, \vec{r}_\perp)}$$

**$q(x, \vec{r}_\perp)$  model** (Burkardt hep-ph/0207047)

# Generalized form factors

$A_{10}, A_{20}, A_{30}$





# Generalized form factor ratios $A_{30} / A_{10}$

GPD parameterization:  
Nucleon form factors,  
CTEQ parton distributions,  
Regge behavior,  
Ansatz

Diehl, Feldmann, Jakob, Kroll EPJC 2005

# Origin of Nucleon Spin

---

# Frames in which to think about spin

---

- Infinite momentum frame
  - No condensates or backward diagrams
  - Simplification of  $A^+ = 0$  gauge
  - Parton distributions
  - Don't know how to solve Light Cone QCD nonperturbatively
- Lab frame
  - Familiar condensates and diagrams
  - Gauge invariant
  - Know how to solve Lattice QCD - Forced to use Lab

# Gauge Invariant Decomposition of Nucleon Spin

X. Ji PRL 78, 610 (1997)

$$J^i = \frac{1}{2} \epsilon^{ijk} \int d^3x [T^{\alpha\nu} x^\mu - T^{\alpha\mu} x^\nu] = J_q^i + J_g^i$$

$$\vec{J}_q = \int d^3x \psi^\dagger [\vec{\gamma}\gamma_5 + \vec{x} \times (-i\vec{D})] \psi$$

$$= \frac{1}{2} [A_{20}(q^2 = 0) + B_{20}(q^2 = 0)]$$

$$\vec{J}_g = \int d^3x [\vec{x} \times (\vec{E} \times \vec{B})]$$

$$\neq \Delta g$$

- $A_{20}$  and  $B_{20}$  are generalized form factors defined below
- Cannot write  $J_g$  as sum of helicity and orbital contributions of local operators

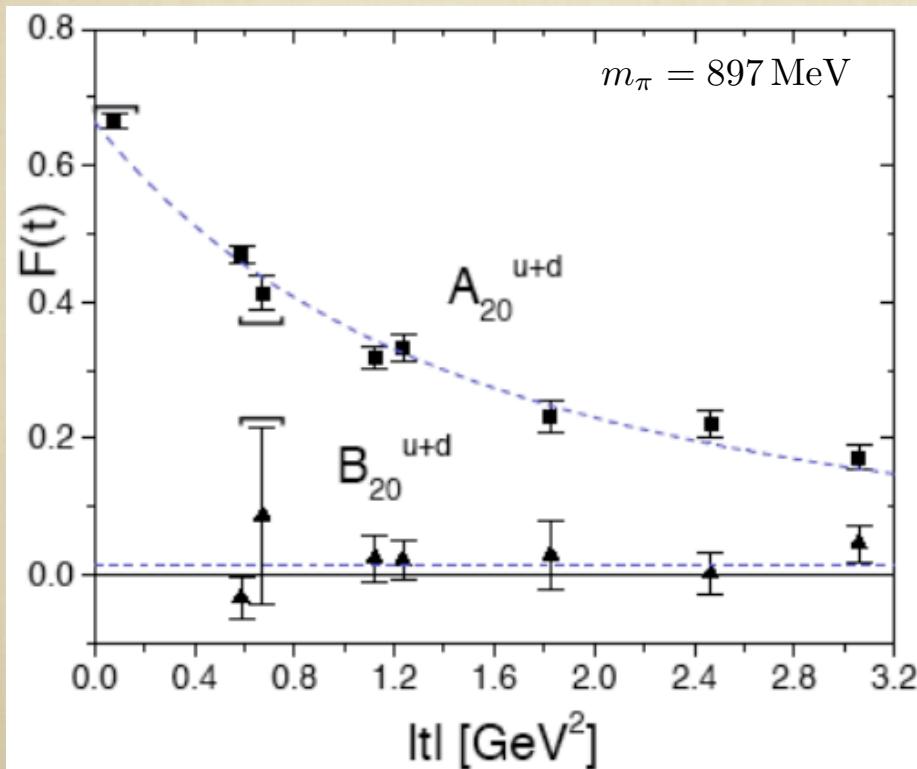
# Quark contributions to proton spin

“Spin crisis” - only  $\sim 30\%$  arises from quark spins

quark spin contribution  $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1 \rangle_{\Delta u + \Delta d} \sim \frac{1}{2}0.682(18)$

total quark contribution (spin plus orbital)

$$J_q = \frac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)] = \frac{1}{2}[\langle x \rangle_{u+d} + B_{20}^{u+d}(0)] \sim \frac{1}{2}0.675(7)$$



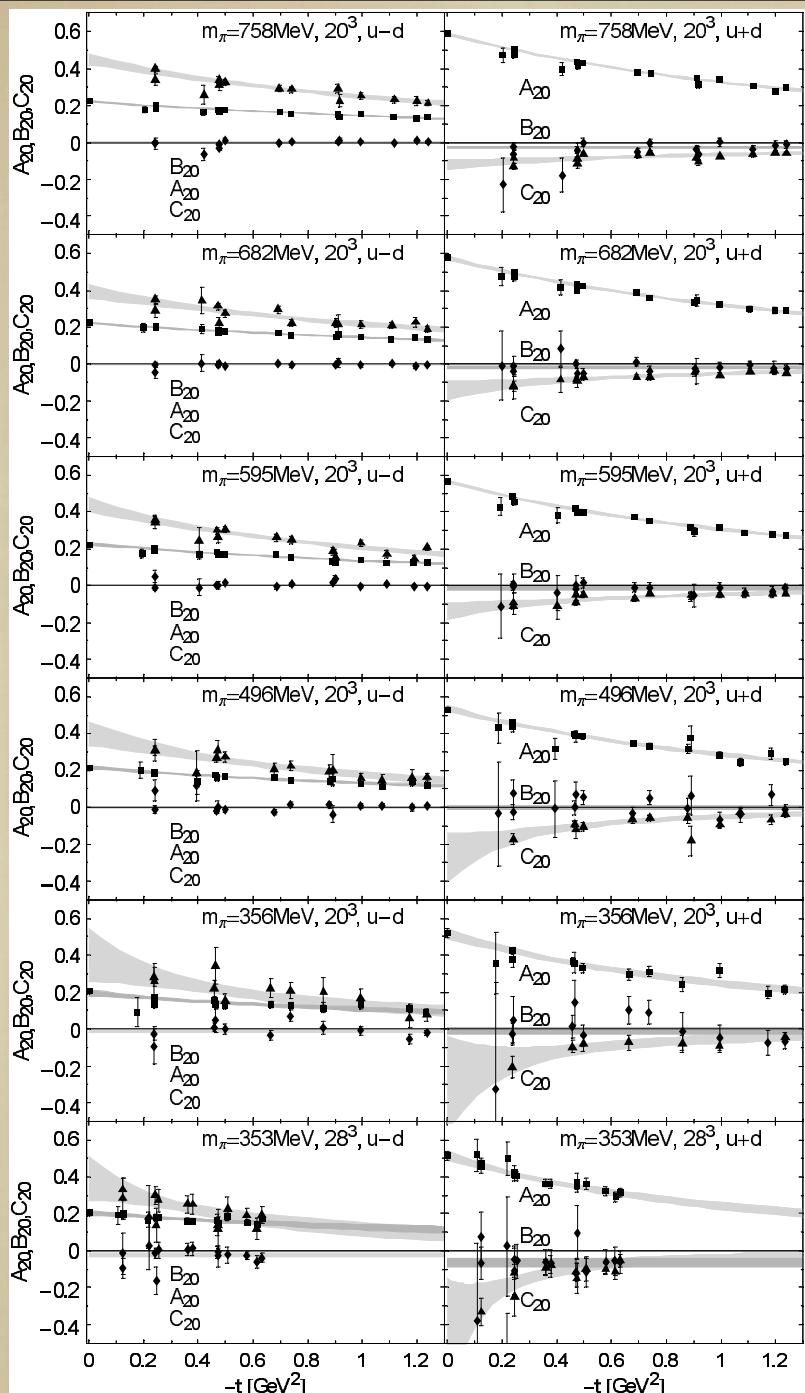
Spin Inventory  
(heavy quarks)  
68% quark spin  
0% quark orbital  
32% gluons

# First x moments:

$A_{20}, B_{20}, C_{20}$

Consistent with large  
N behavior [Goeke et. al.]

$$\begin{aligned} |A_{20}^{u+d}| &> |A_{20}^{u-d}| \\ |B_{20}^{u-d}| &> |B_{20}^{u+d}| \\ |C_{20}^{u+d}| &> |C_{20}^{u-d}| \end{aligned}$$



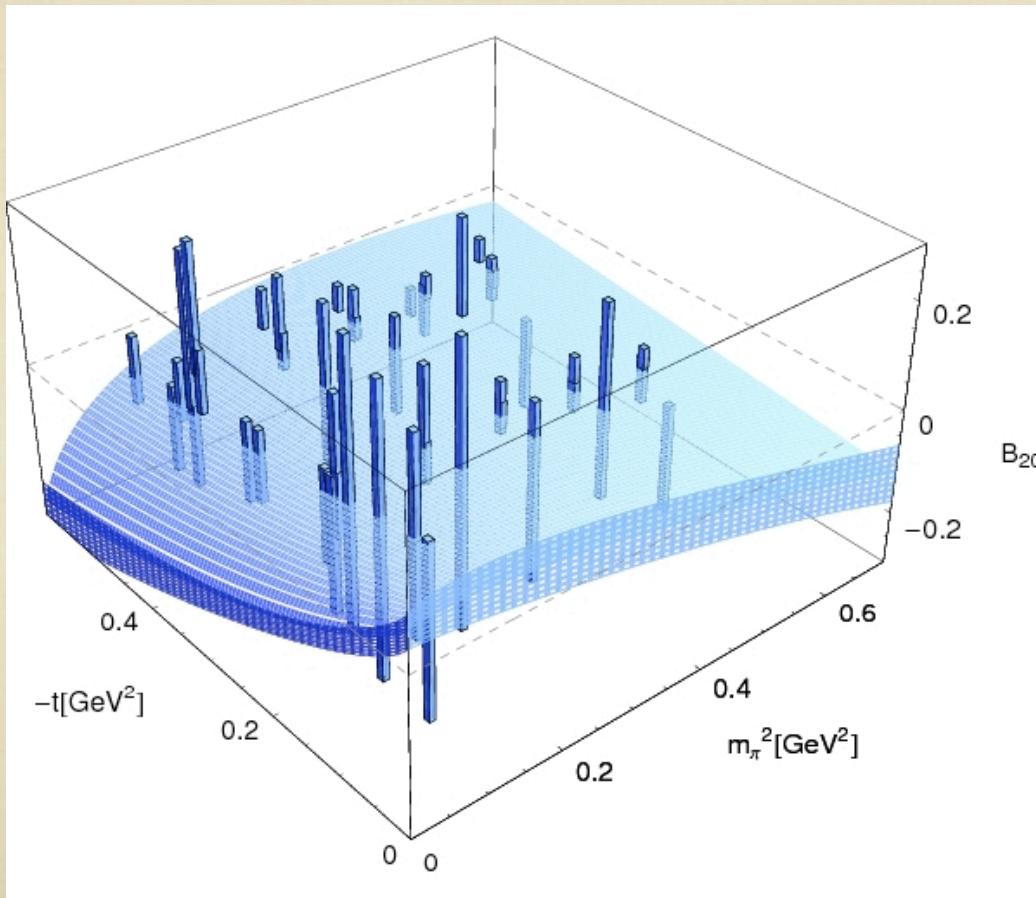
# Chiral Extrapolation of $B_{20}^{u+d}(t, m_\pi)$

Chiral extrapolation  $\mathcal{O}(p^2)$  CBChPT + $\mathcal{O}(p^3)$  corrections

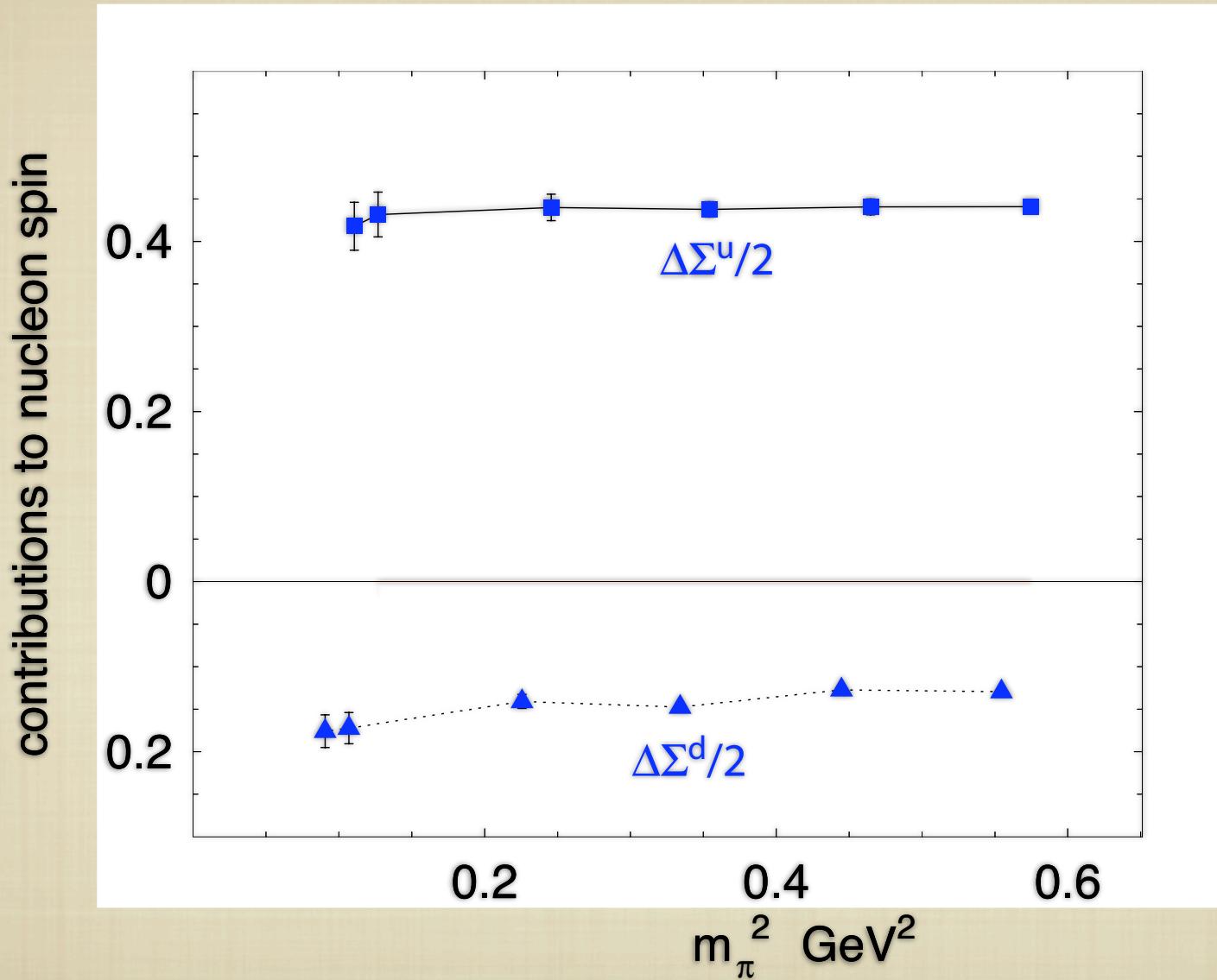
Note: connected diagrams only

(Dorati, et. al.)

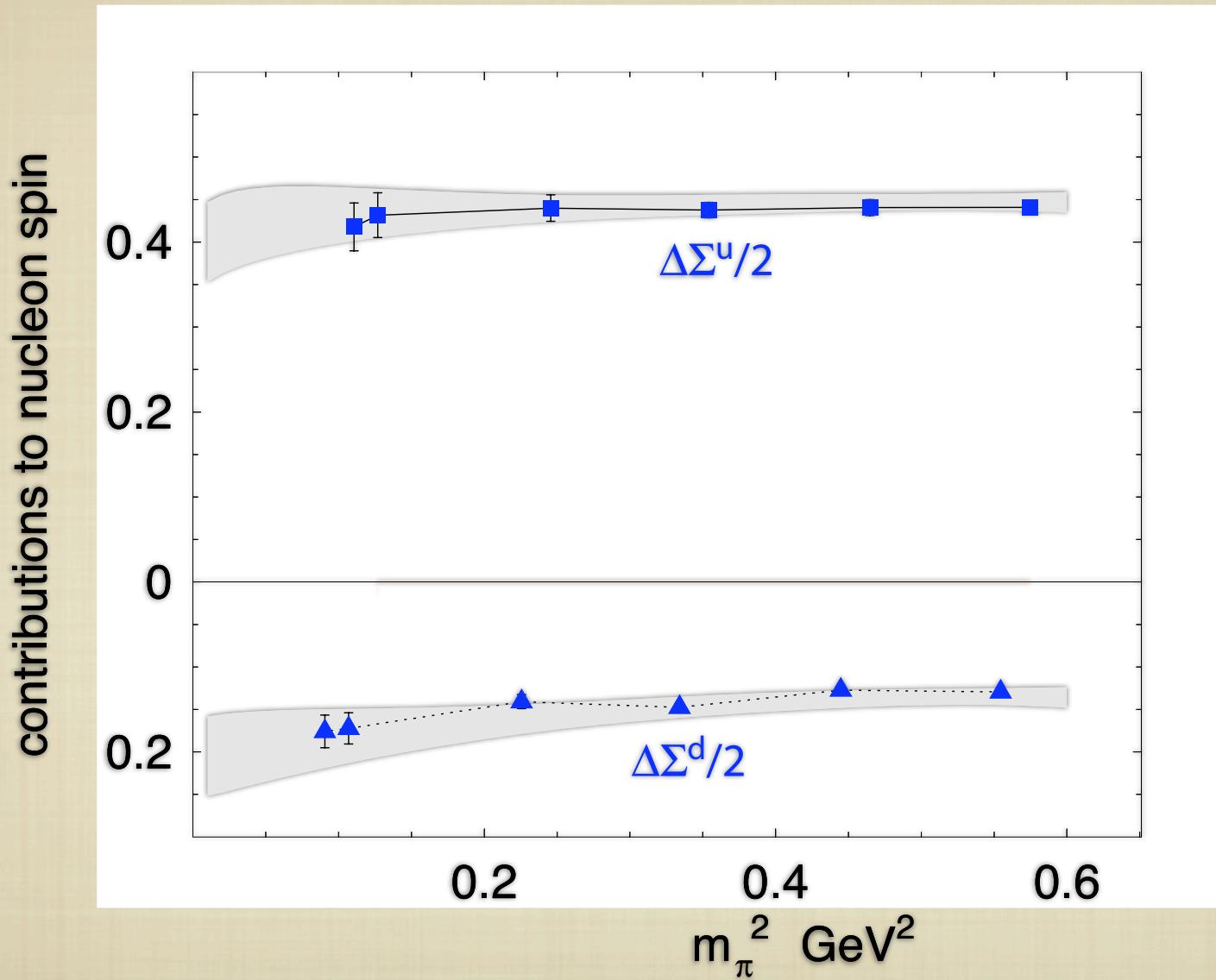
$$B_{20}^{u+d}(t, m_\pi) = A_{20}^{0,u+d} h_B^{u+d}(t, m_\pi) + \Delta B_{20}^{t,u+d}(t, m_\pi) + \frac{m_N(m_\pi)}{m_N} \left\{ B_{20}^{0,u+d} + \delta_B^t t + \delta_B^{m_\pi} m_\pi^2 \right\} \dots$$



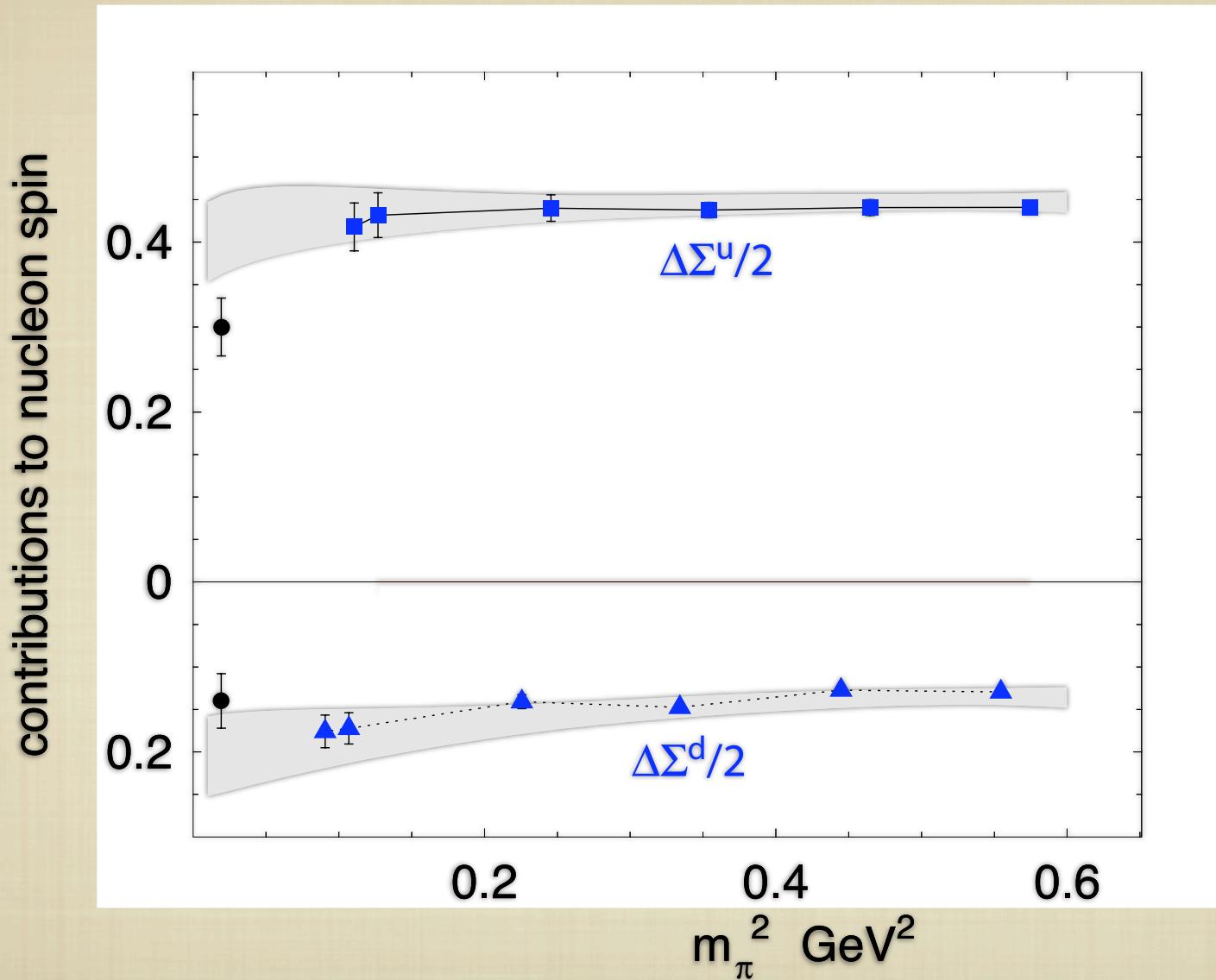
# Quark contributions to the proton spin



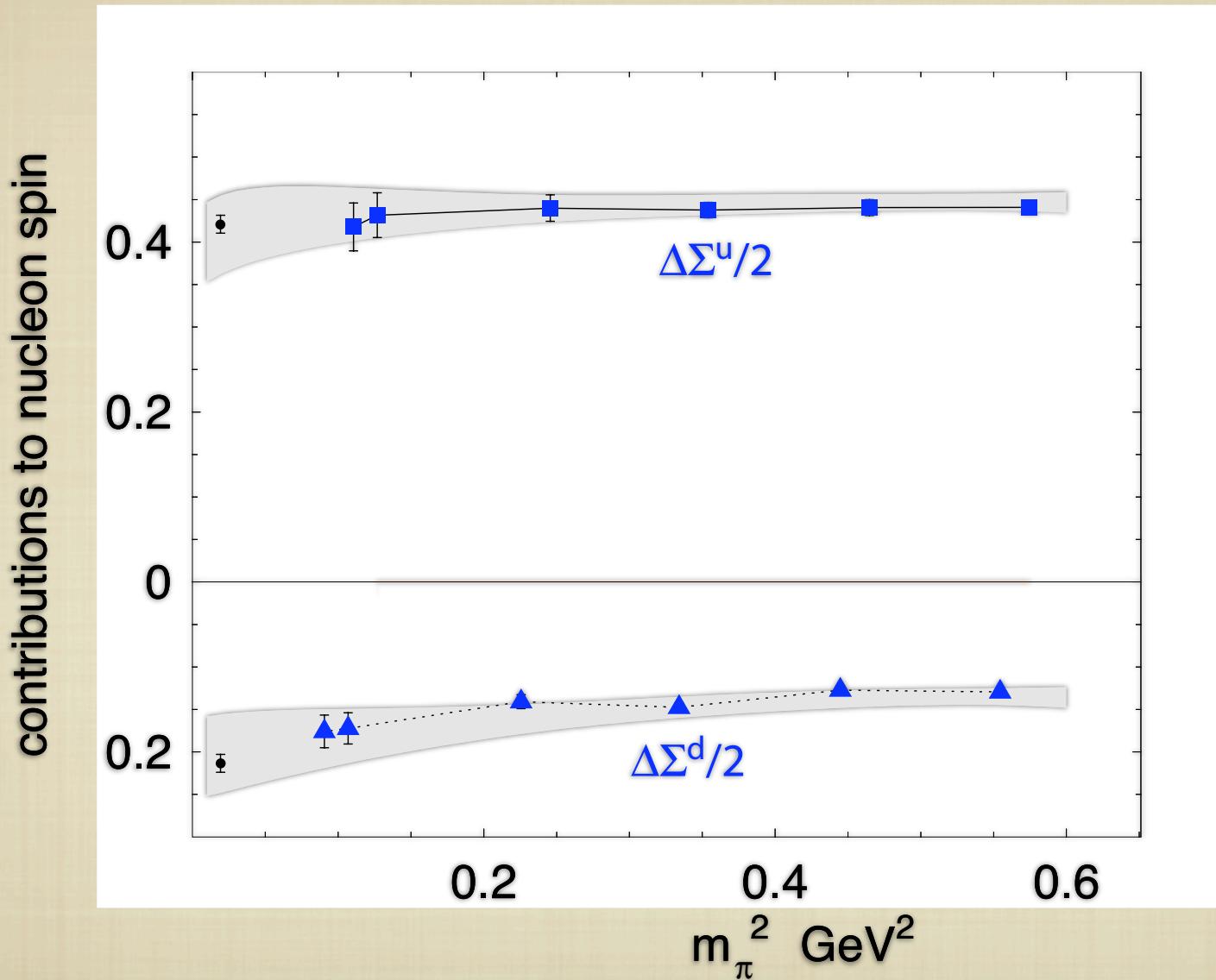
# Quark contributions to the proton spin



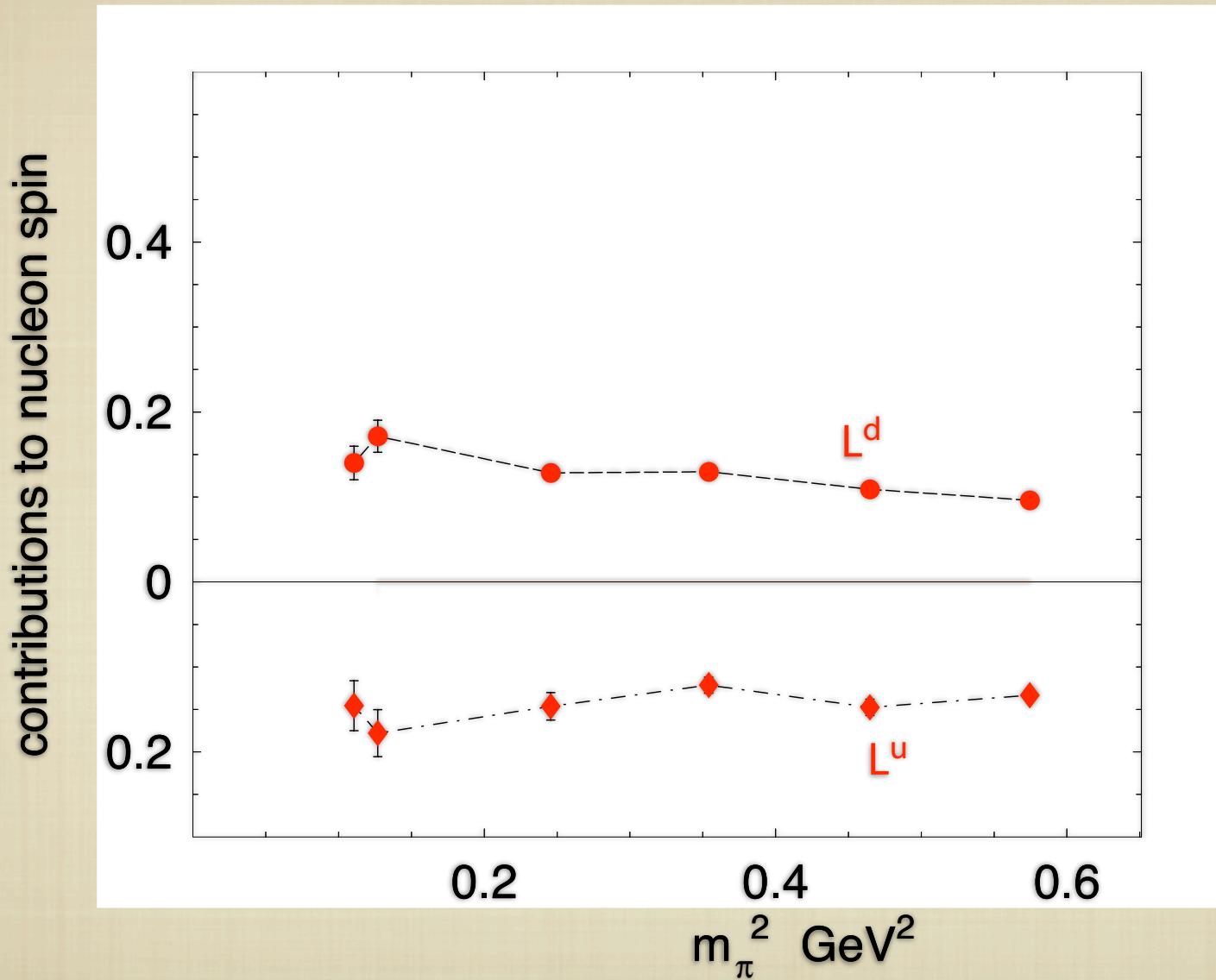
# Quark contributions to the proton spin



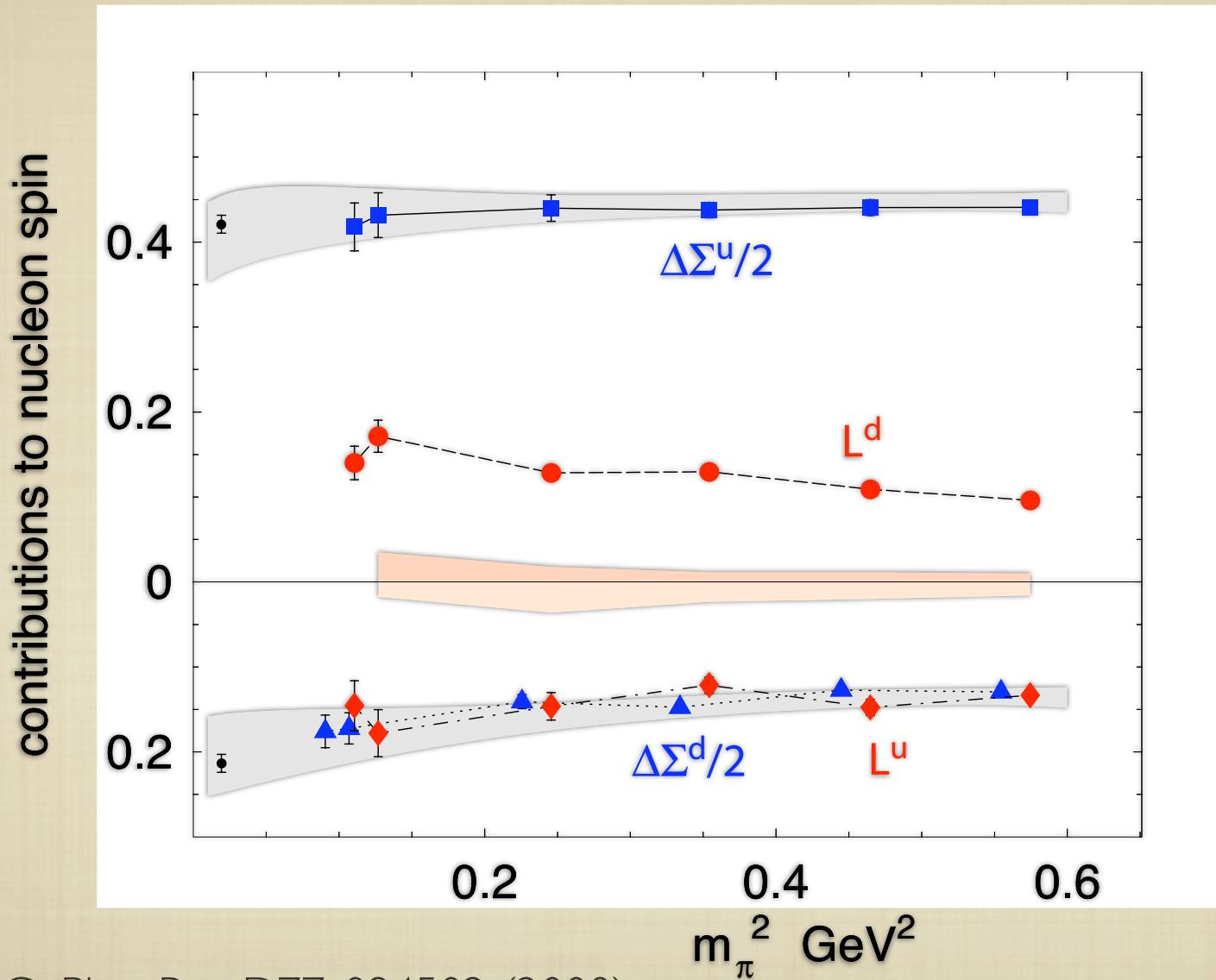
# Quark contributions to the proton spin



# Quark contributions to the proton spin



# Quark contributions to the proton spin



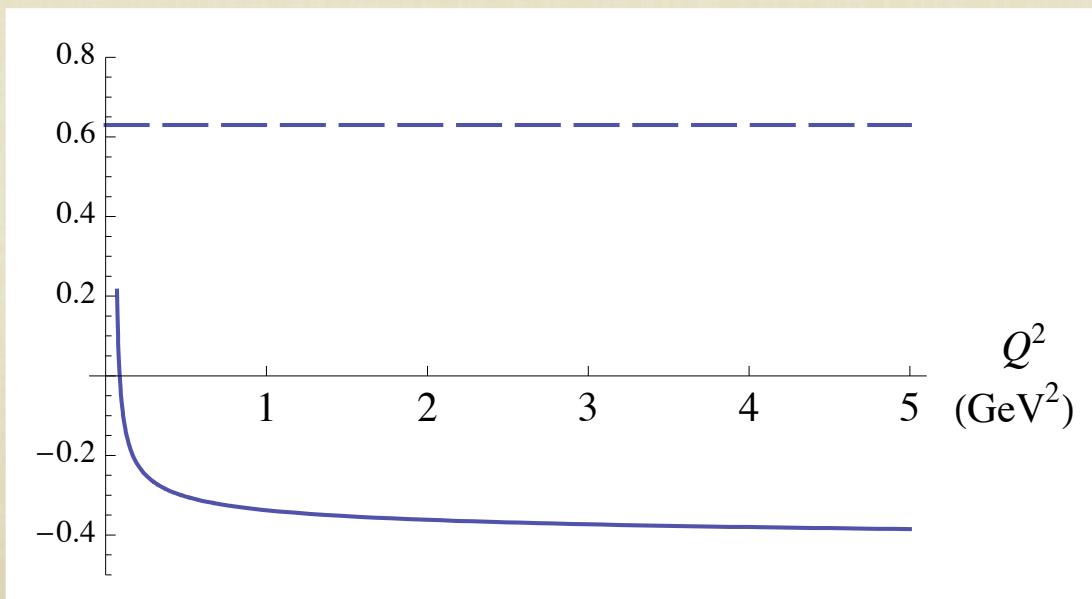
# Evolution of nonsinglet angular momentum

Nonsinglet J has simple evolution

A.W.Thomas arXiv:0803.2775 [hep-ph]

Spin conserved, so large change in L

$$L^{u-d}(t) + \frac{\Delta\Sigma^{u-d}}{2} = \left(\frac{t}{t_0}\right)^{-\frac{32}{81}} \left(L^{u-d}(t_0) + \frac{\Delta\Sigma^{u-d}}{2}\right) \quad t = \ln\left(\frac{Q^2}{\Lambda_{QCD}^2}\right)$$



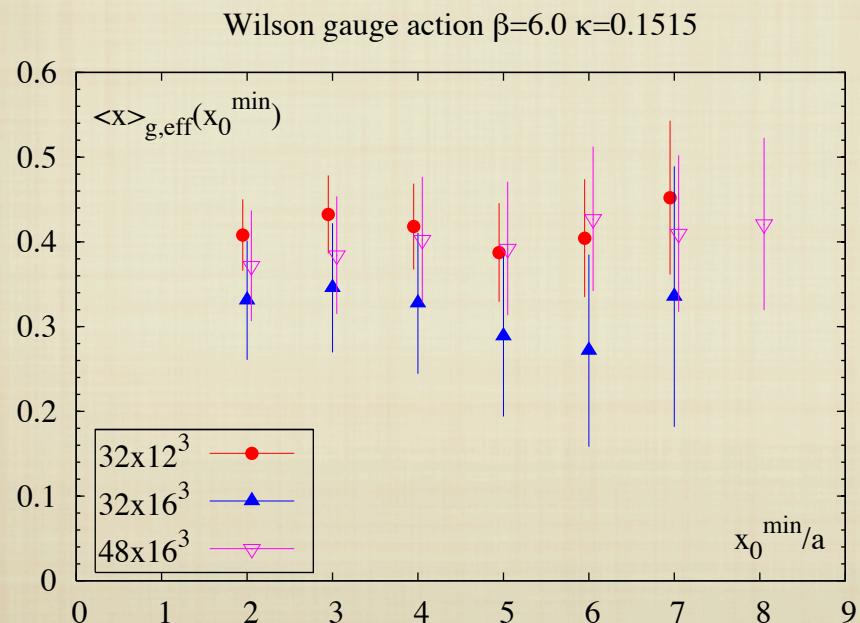
# Proton Spin

---

- Current status:
  - ~41 % from up and down quark spin
  - ~ 0 % from up and down quark orbital angular mom.
  - Remainder in heavier quarks and in glue
- Current effort:
  - Disconnected quark diagrams
  - Direct calculation of gluon contribution
    - First step - Gluon contribution to pion momentum

# Gluon momentum fraction in pion

- Notoriously difficult: 5000 configurations - no signal
- Improved operator  $E^2 - B^2$  (Signal improved  $\times 40$ )
- Normalize operator by ratio of entropy at finite T



Harvey Meyer and J.N.  
PRD 77 037501 (2008)

# Gluon momentum fraction in pion

---

- Mixing with quarks: perturbative correction

$$\begin{bmatrix} \bar{T}_{00}^g(\mu) \\ \bar{T}_{00}^f(\mu) \end{bmatrix} = \begin{bmatrix} Z_{gg} & 1 - Z_{ff} \\ 1 - Z_{gg} & Z_{ff} \end{bmatrix} \begin{bmatrix} \bar{T}_{00}^g(g_0) \\ \bar{T}_{00}^f(g_0) \end{bmatrix}$$

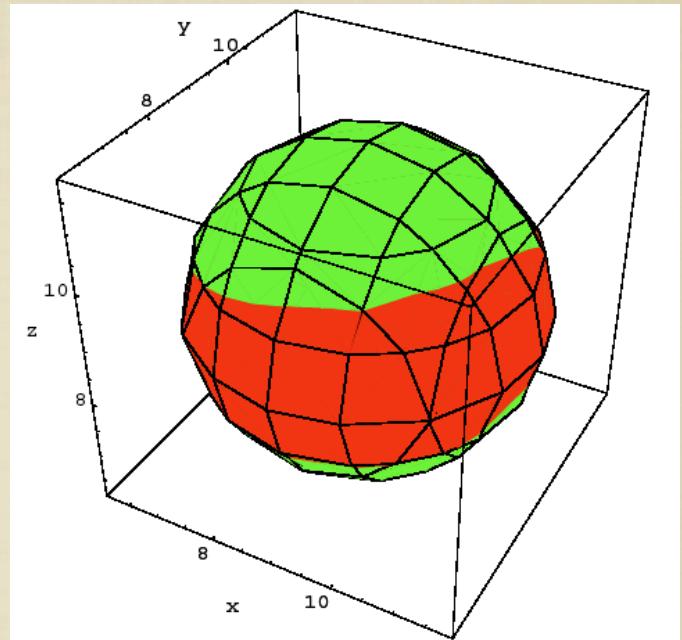
- $\langle x \rangle_{\text{glue}} (\mu = 2 \text{ GeV}) = 0.37 \pm 8 \pm 12$
- Check:  $\langle x \rangle_{\text{glue}} + \langle x \rangle_{\text{quarks}} = 0.99 \pm 8 \pm 12$
- Can also calculate mass from trace anomaly  $E^2 + B^2$   
(need chiral fermions to avoid mixing)

# Baryon shapes

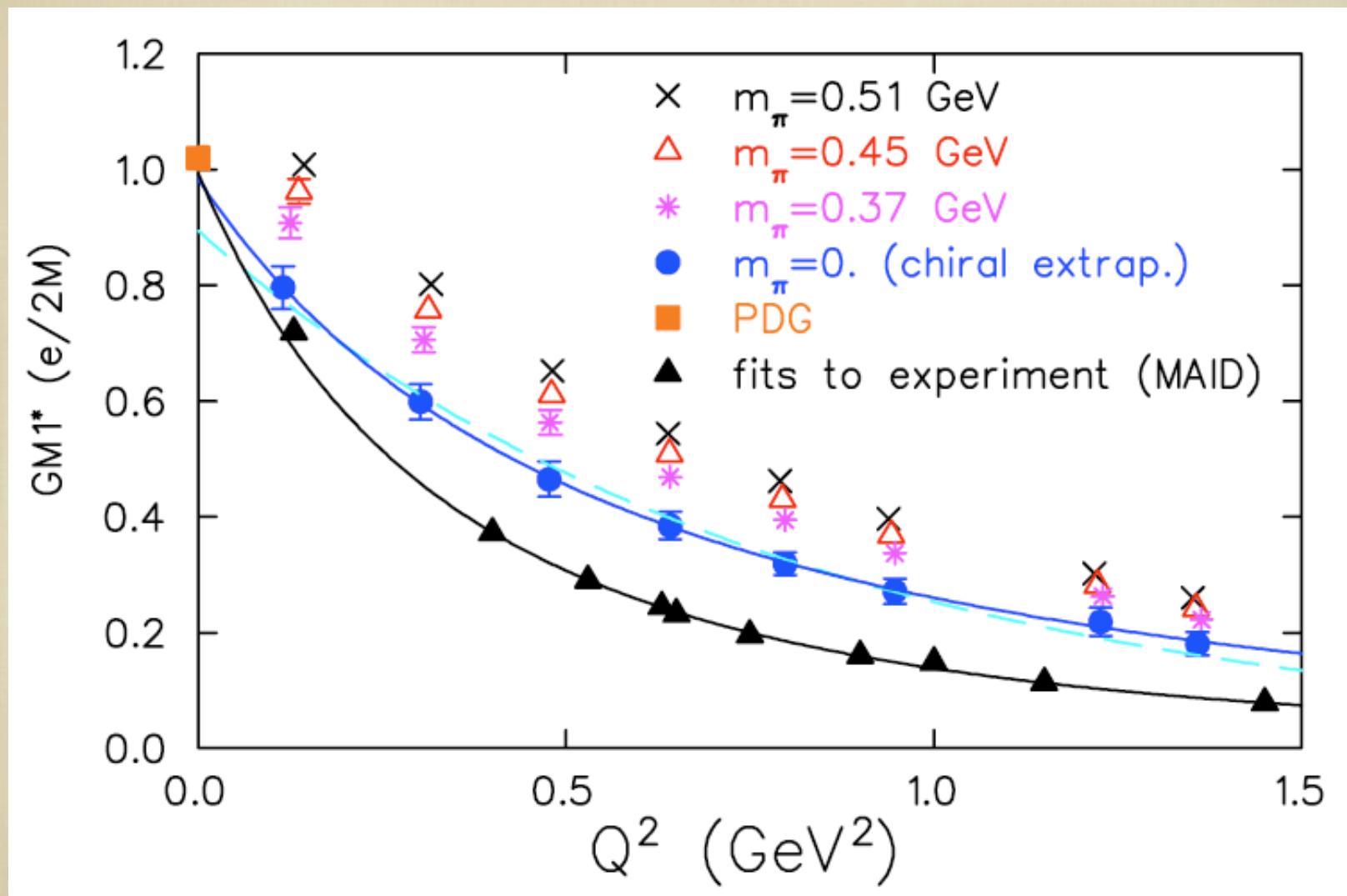
---

# Baryon shapes

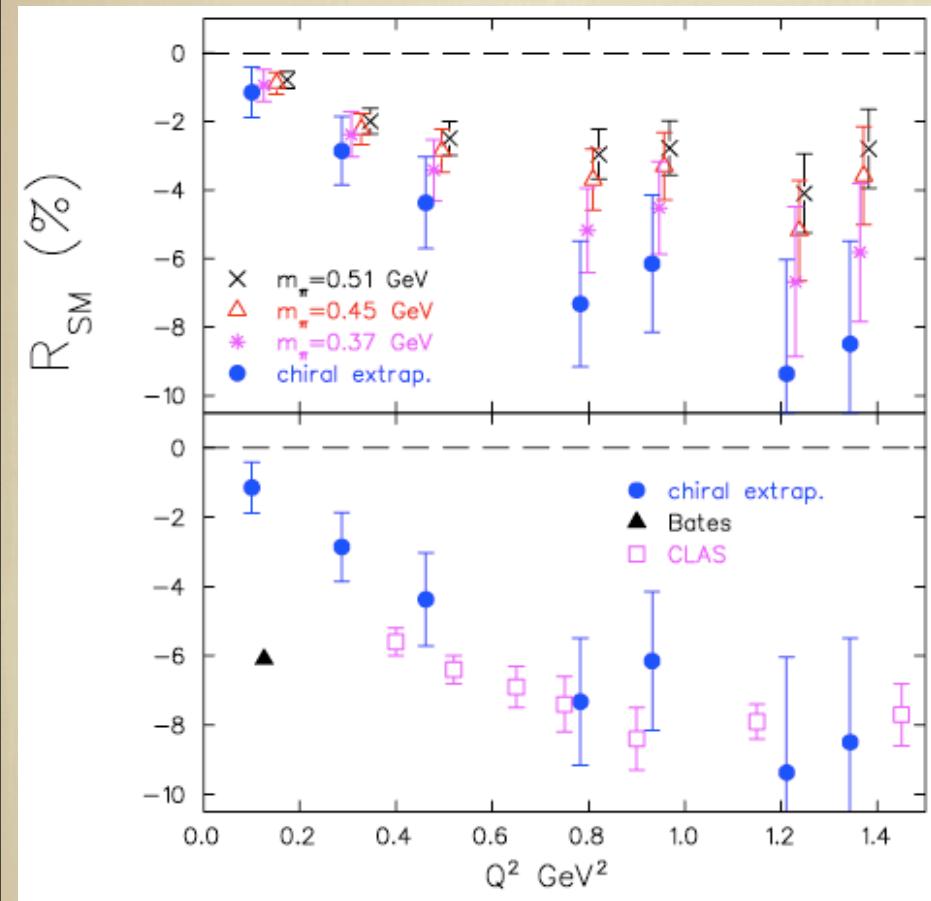
- Observe oblate deformation of spin  $3/2 \Delta$  directly on lattice from density-density correlation function  
(Alexandrou, nucl-th/0311007 )
- Infer deformation experimentally from  $N \rightarrow \Delta$  transition form factor
  - Dominant transition M1
  - $C_2$  and  $E_2$  would vanish if nucleon and  $\Delta$  spherical



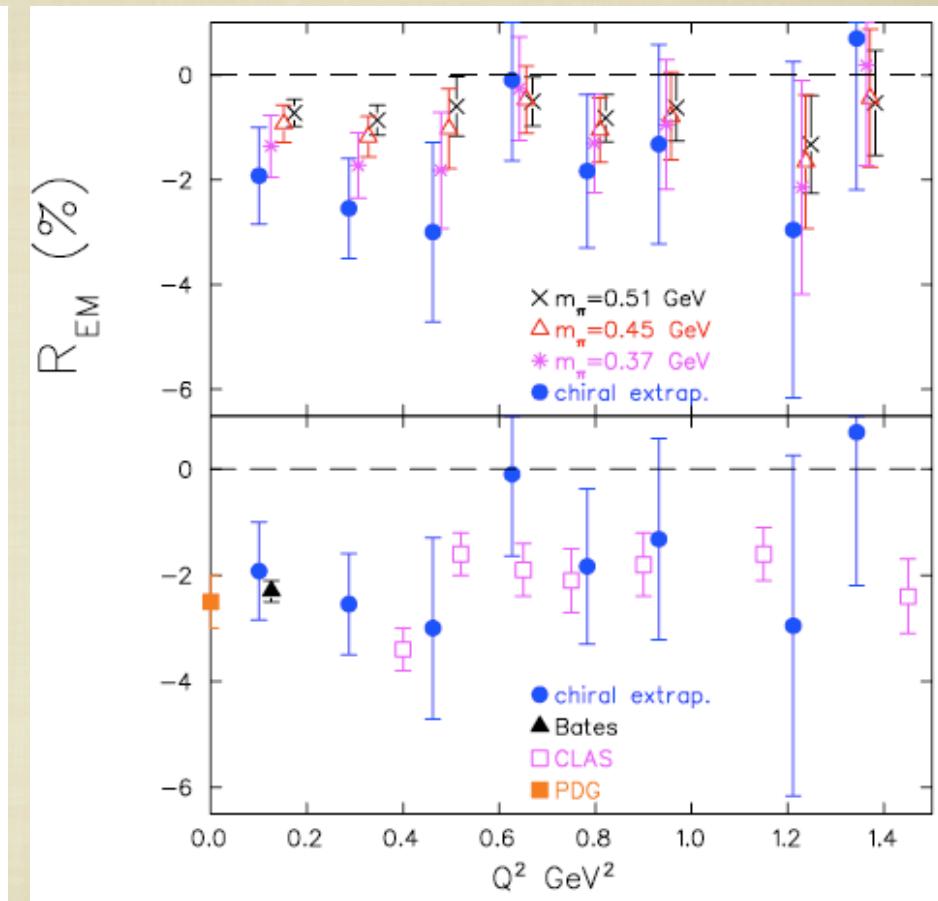
# MI form factor



# Electric and Coulomb transitions



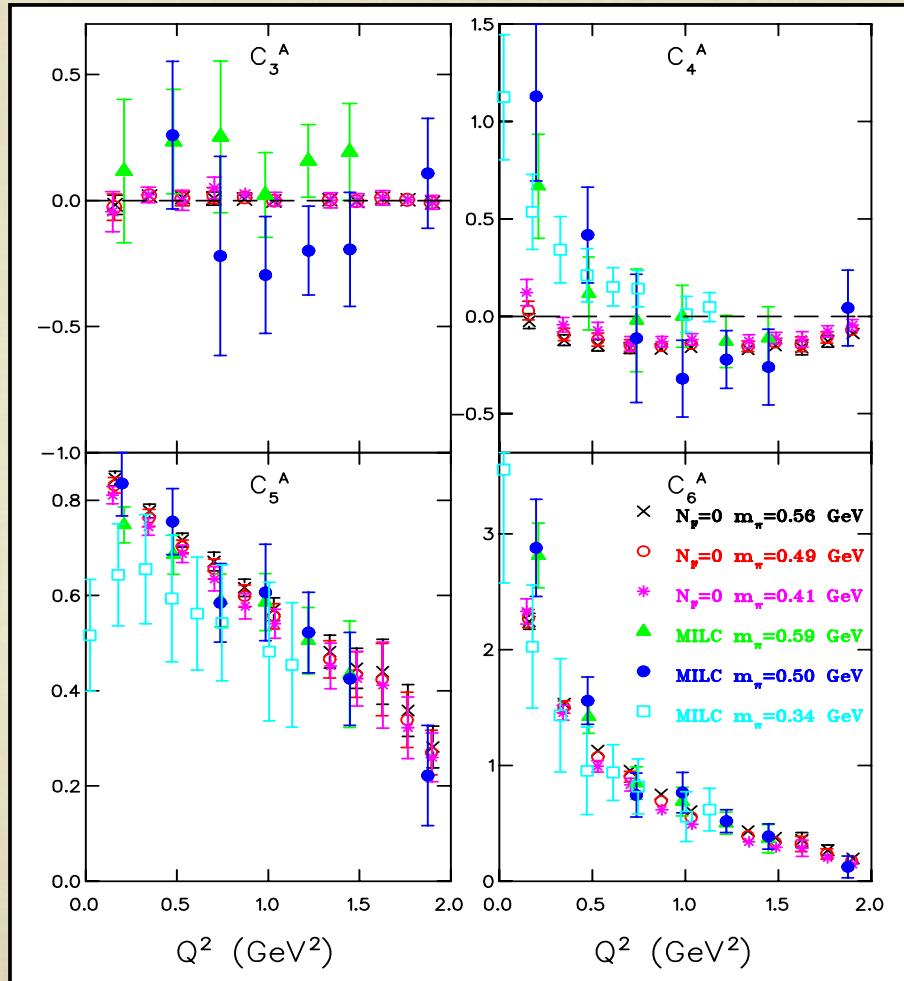
C2/MI



E2/MI

# Axial N-Delta transition form factors

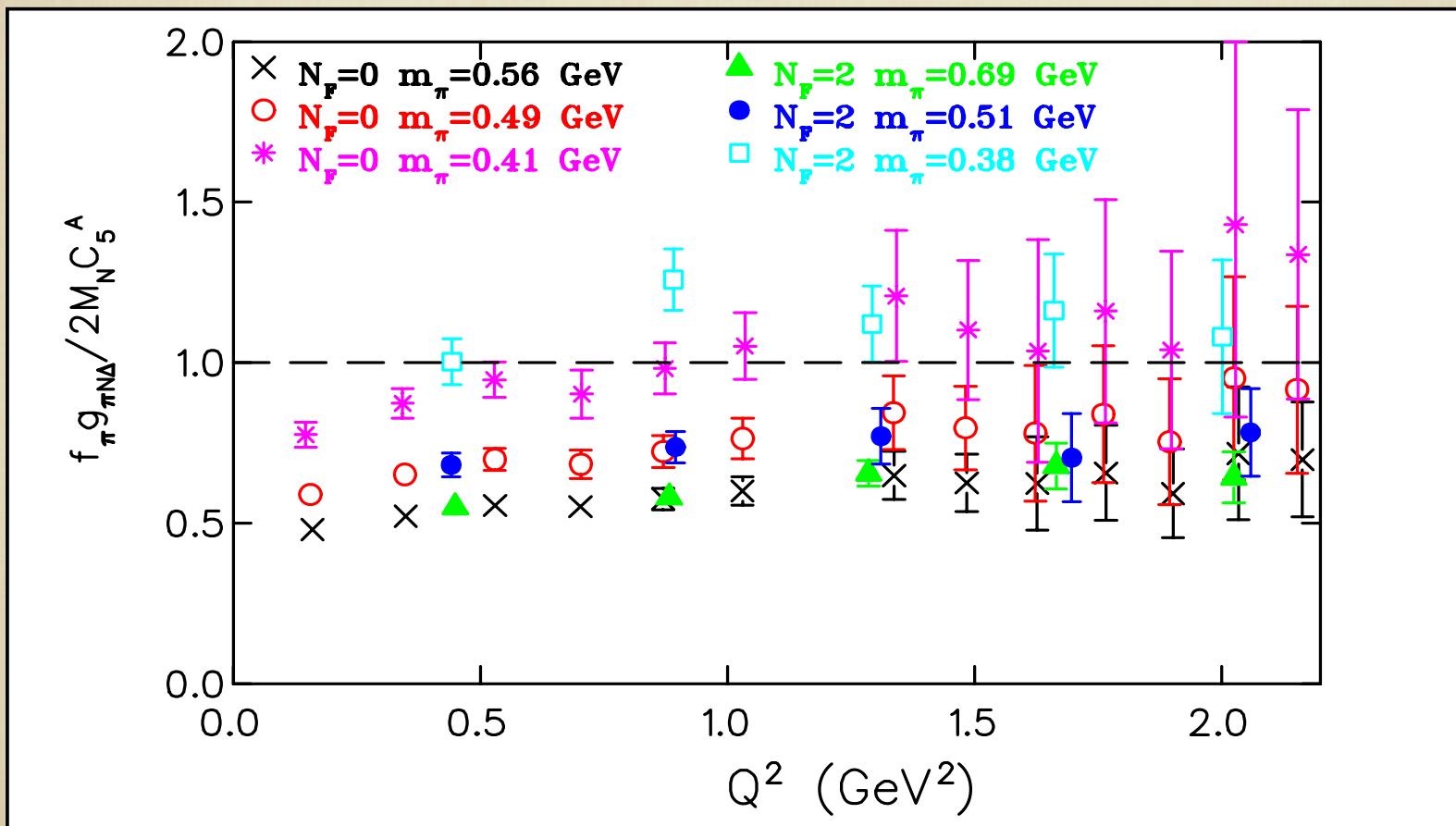
$$\langle \Delta(p', s') | A_\mu | N(p, s) \rangle \propto \bar{u}^\lambda(p', s') \left[ \left( \frac{C_3^A(q^2)}{M} \gamma^\nu + \frac{C_4^A(q^2)}{M^2} p'^\nu \right) (g_{\lambda\mu} g_{\rho\nu} - g_{\lambda\rho} g_{\mu\nu}) q^\rho + C_5^A(q^2) g_{\lambda\mu} + \frac{C_6^A(q^2)}{M^2} q_\lambda q_\mu \right] u(p, s)$$



# Axial N-Delta transition form factors

Off-diagonal Goldberger-Treiman relation

$$C_5^A(q^2) = \frac{F_\pi g_{\pi N \Delta}(q^2)}{2M}$$

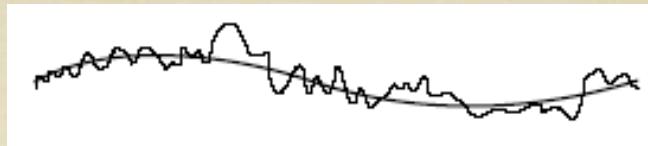


# Insight into how QCD works

---

# Insight into how QCD works: classical solutions

- Stationary phase approximation



$$\int D[A] e^{-\int d^4x S[A]} \sim [\det S''']^{-1} e^{-\int d^4x S[A_{cl}]}$$

- Instanton solutions connect vacua with different winding numbers

$$A_\mu^a(x) = \frac{2\eta_{a\mu\nu}x_\nu}{x^2 + \rho^2}$$

$$S = \frac{1}{4} \int F^2 = \frac{8\pi^2}{g^2}, \quad Q = \frac{q^2}{32\pi^2} \int F \tilde{F} = 1$$

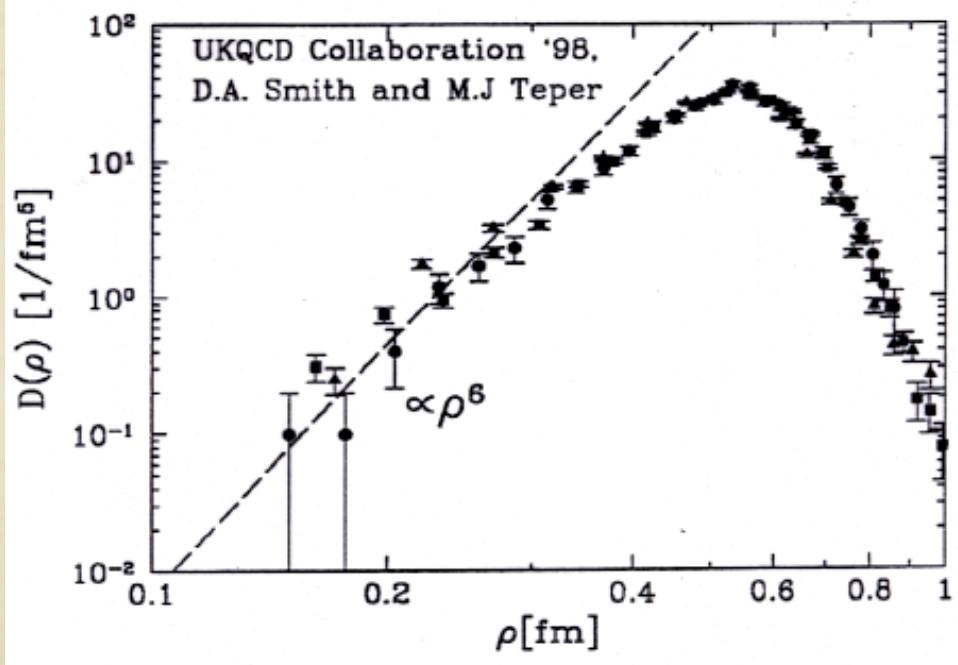
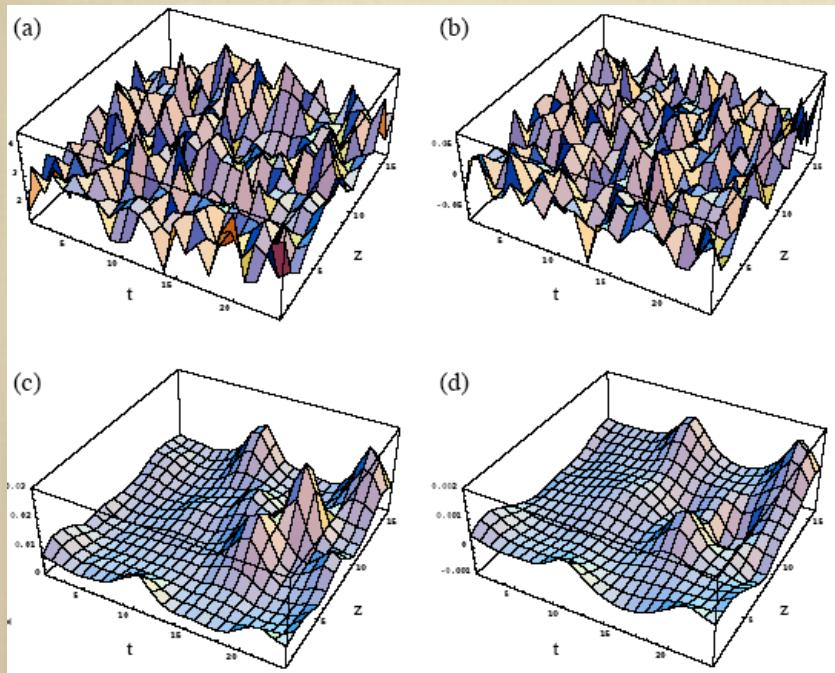
- To what extent are analytic expectations observed on lattice?

# Instantons on the lattice

- Cooling (relaxation) reveals lumps with  $S \sim \frac{8\pi^2}{g^2}$  and  $Q \sim \pm 1$
- For small size  $\rho$ , distribution  $\propto \rho^6$

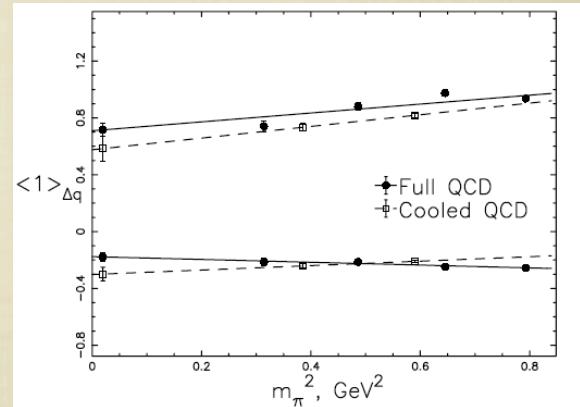
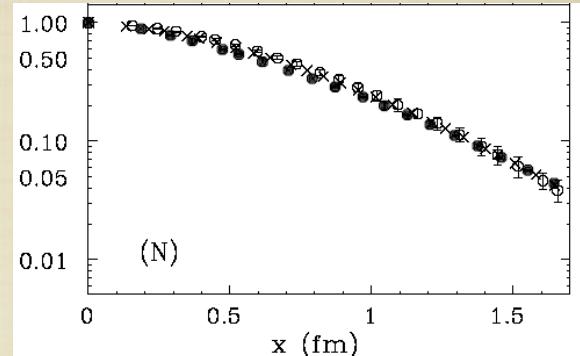
$s(x)$

$q(x)$



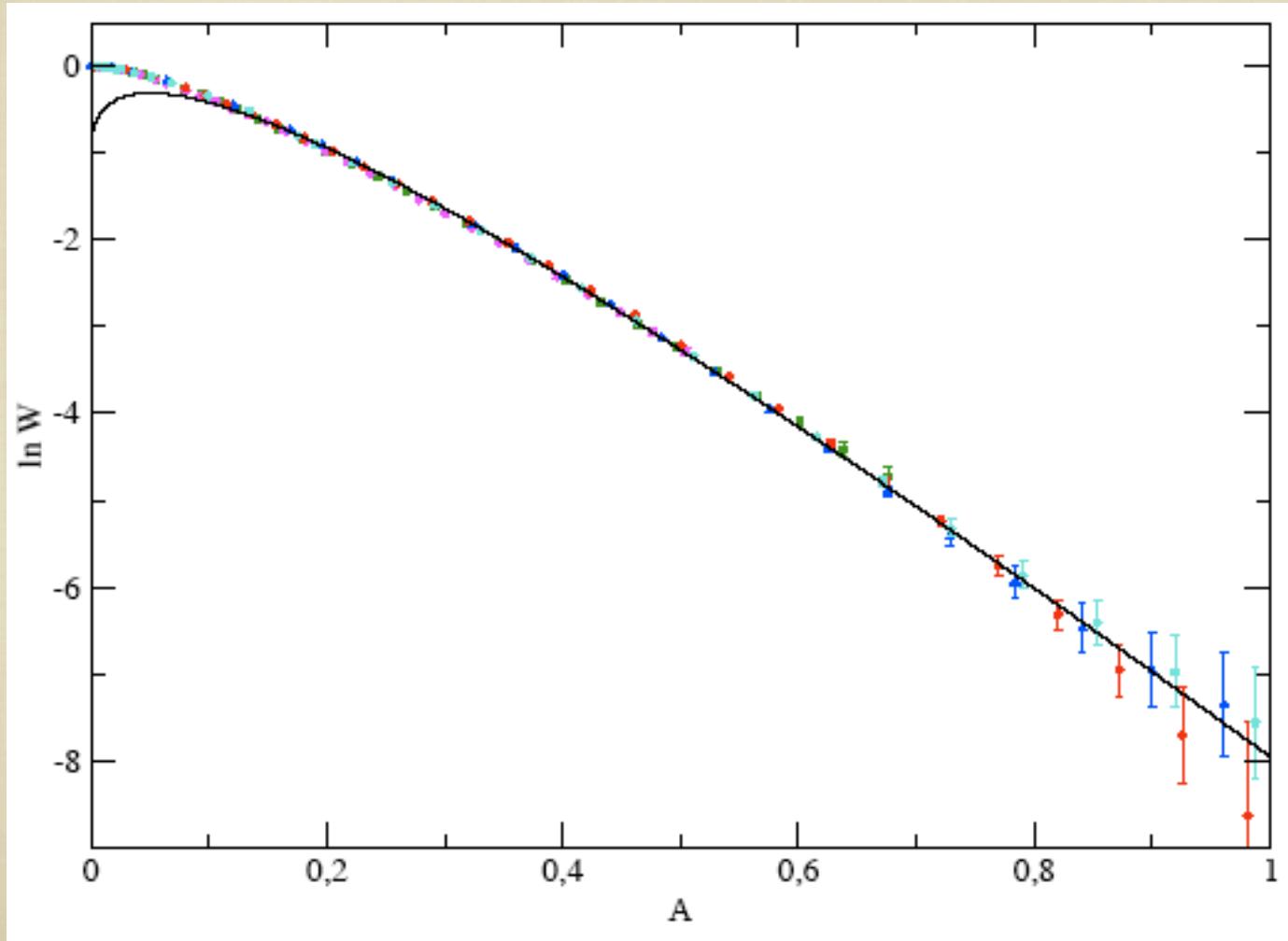
# Instantons on the lattice

- Observables calculated with only instantons close to those including all gluons
- Observe quark zero modes localized at instantons
- Zero modes from instantons generate and dominate light quark propagators
- Topological susceptibility from instantons,  $X=(180\text{MeV})$ , yields  $\eta'$  mass



# Confinement from instantons

Ensemble of regular gauge instantons yields area law hep-th/0306105

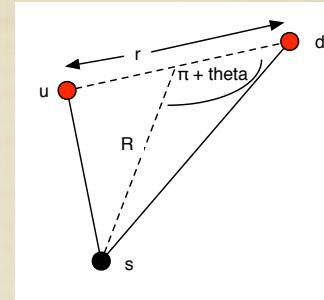


# Diquark correlations in heavy light light baryon

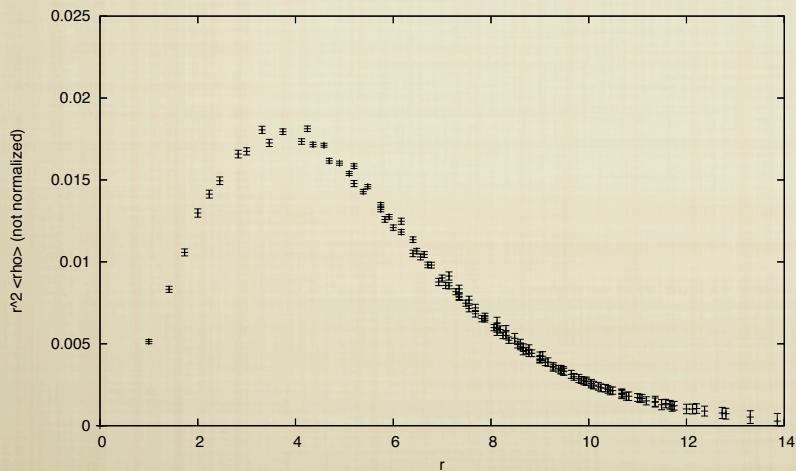
Good diquarks:  
color antitriplet  
flavor antisymmetric  
spin singlet

$$(u C \gamma_5 d) h$$

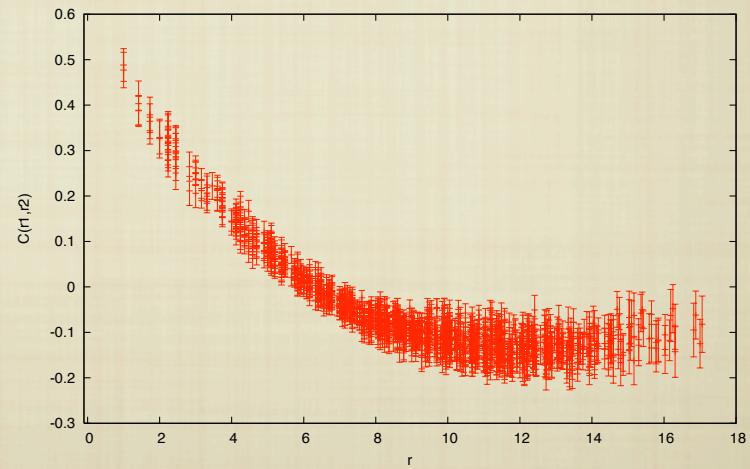
Patrick Varilly - senior thesis



$$\langle \rho(r) \rangle$$



$$C(r_1, r_2) = \frac{\langle \rho(r_1) \rho(r_2) \rangle - \langle \rho(r_1) \rangle \langle \rho(r_2) \rangle}{\langle \rho(r_1) \rangle \langle \rho(r_2) \rangle}$$



# Summary and Challenges

---

# Summary

---

- Lattice Field theory has become powerful tool to solve QCD and understand hadronic physics
  - Substantial theoretical issues and accomplishments
  - Resources now available to solve frontier problems
- Entering era of quantitative solution in chiral regime
  - Moments of quark distributions
  - Form factors:  $F_1, F_2, G_A, G_P, N \rightarrow \Delta$
  - Generalized form factors A B C
    - Transverse structure
    - Origin of nucleon spin
  - Insight: instantons, diquarks, dependence on parameters
- Two other major areas:
  - QCD thermodynamics
  - Weak decays

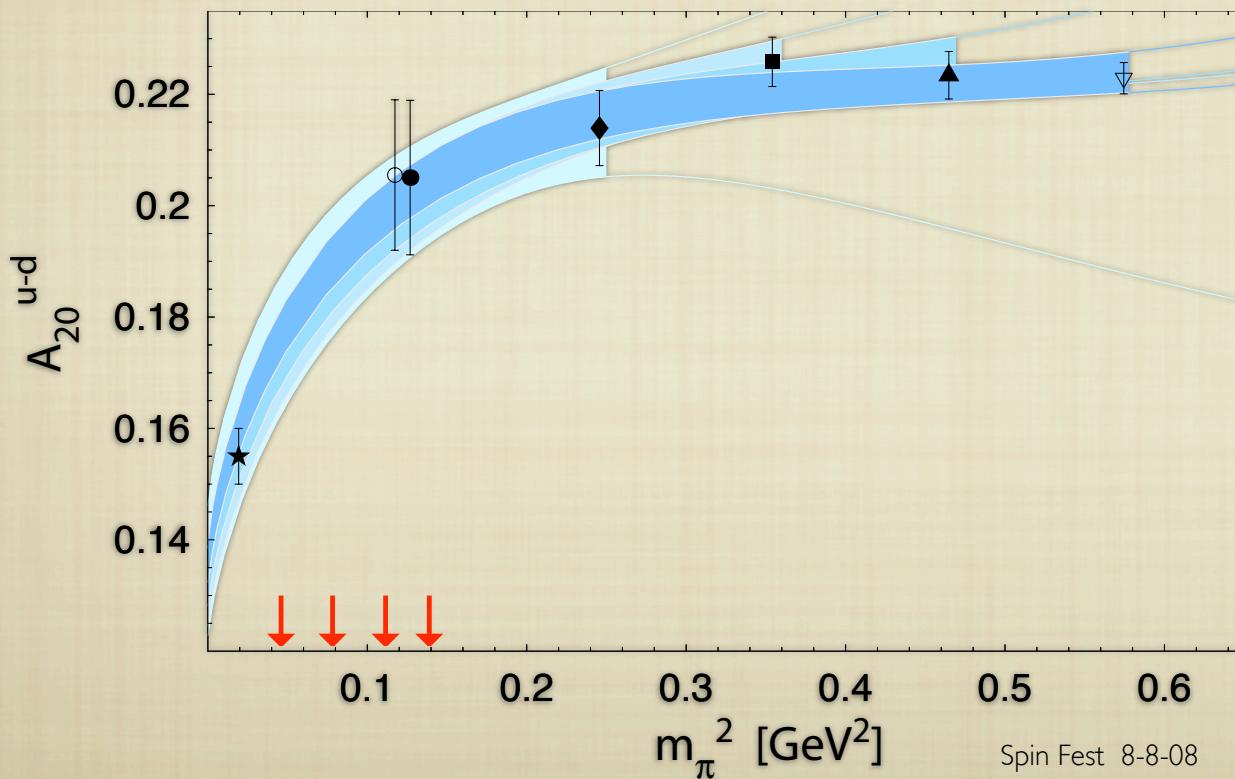
# Current effort and future challenges

---

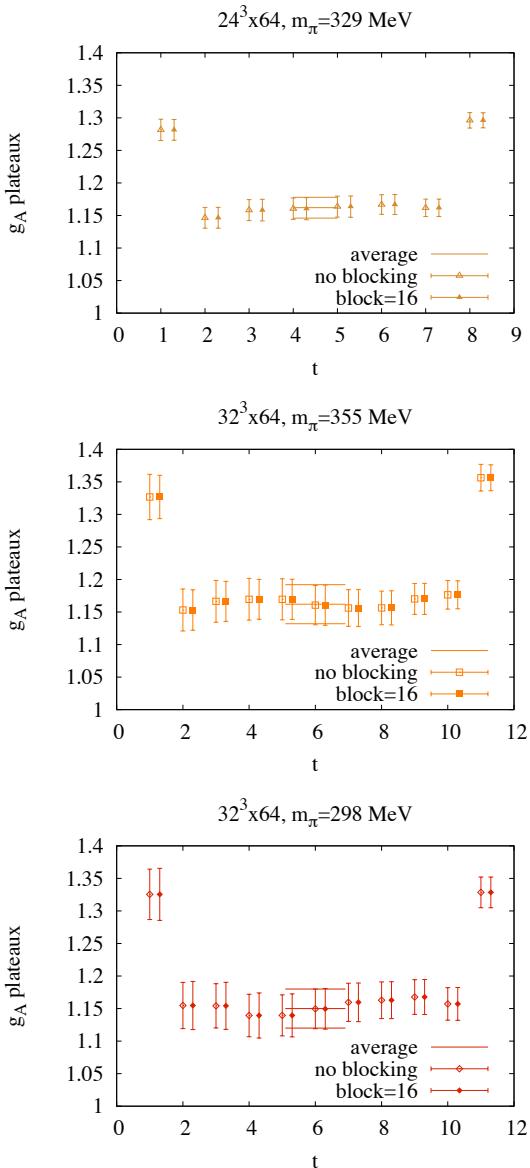
- Chiral fermions down to physical pion mass
- Flavor singlet observables
  - Disconnected diagrams
    - Calculate proton and neutron separately, not just difference
    - Strangeness content of nucleon
  - Gluon observables
    - Contribution to mass, momentum, spin
    - Quark and gluon mixing, evolution
- Role of diquarks in hadrons
- Unstable states

# Current Dynamical DW Calculations

- Collaboration of LHPC, RBC, UKQCD
- $a = 0.086 \text{ fm}$
- $m_\pi = 371, 330, 276, \text{ and } 206 \text{ MeV}$
- $L = 2.8, 2.8, 2.8, \text{ and } 4.1 \text{ fm}$

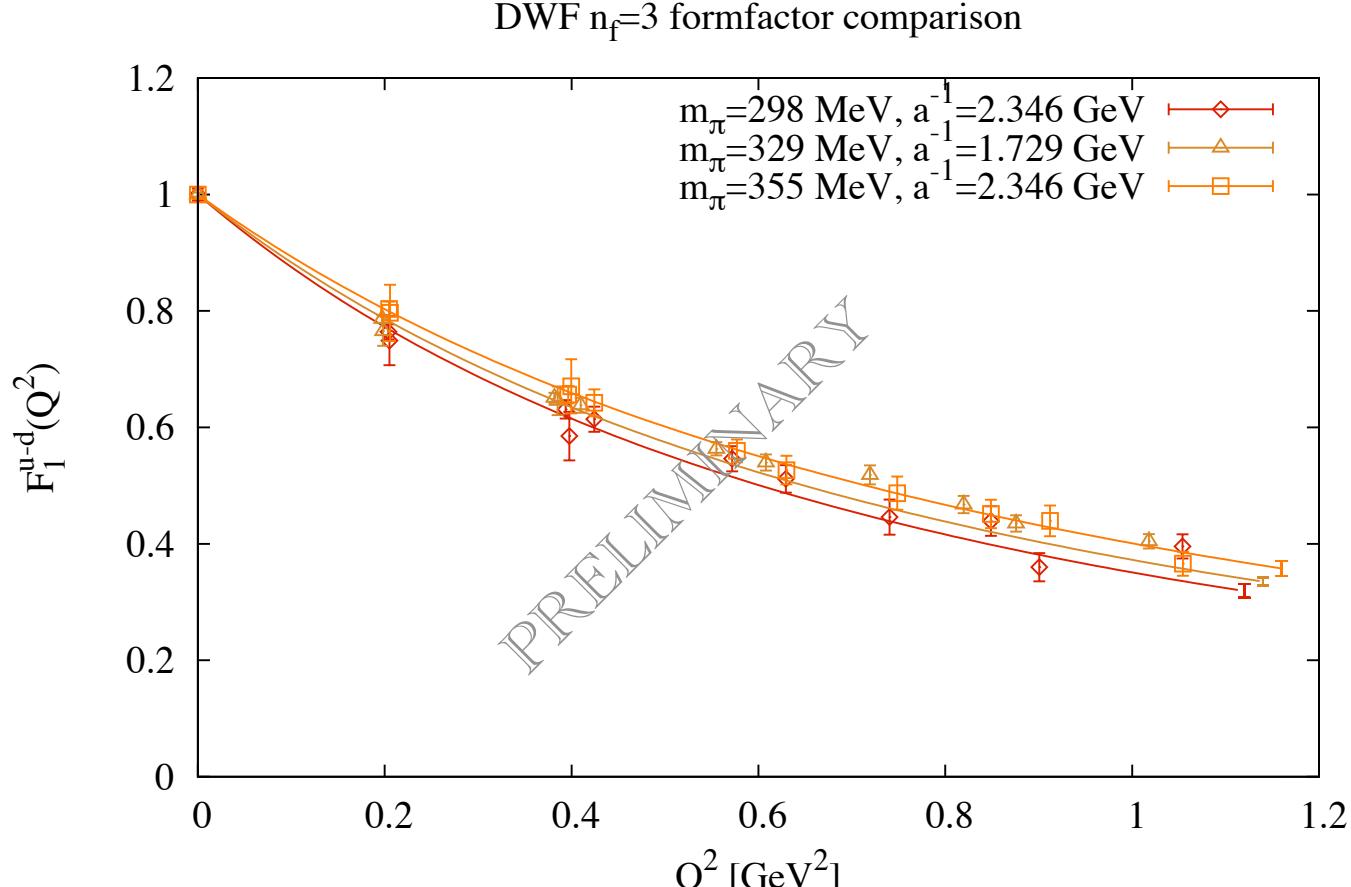


# Preliminary Domain Wall Results



Plateaus for axial charge  
with dynamical domain  
wall fermions

# D.W Fermion Form Factors



Dipole fit in range  $0 \leq Q^2 \leq 0.4 \text{ GeV}^2$